Solving Word Problems

An Ineffective Approach

A common approach when teaching students to solve word problems is highlighting key words and assigning a mathematical operation to the word(s). For example, **"take away"** means subtraction needs to be used to solve the problem. Attributing operations to key words can be mathematically incorrect. In the following two examples a key word approach **does not work**.

"Josh had some toy trains. He <u>took away</u> five and <u>gave</u> them to his friend. He was <u>left</u> with three trains. How many trains did he have to start with?"

In this problem words such as **"took away"**, **"gave"** and **"left"** relate to addition.

The answer is found by

$$5 + 3 = \boxed{8}$$

"Mira has a collection of 23 toy figurines. This is 7 <u>more</u> than Kali. Write what you would enter into a calculator to find out how many toy figurines Kali has?"

In this problem the word **"more"** relates to subtraction.

The answer is

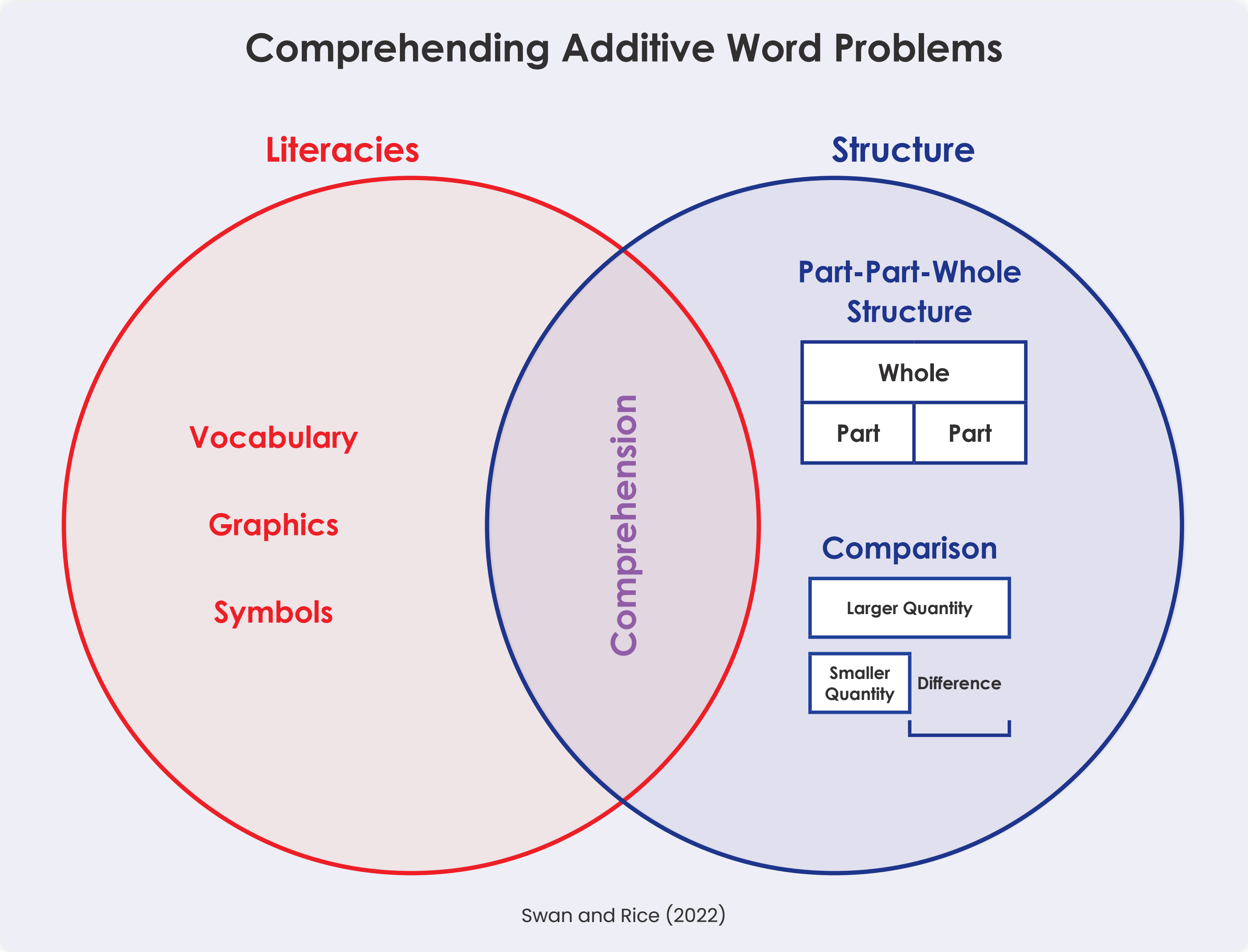
$$23 - 7 = \boxed{}$$

Powell and Fuchs (2018) state "Neither of these approaches – defining problems in terms of a single operation or linking key words to specific operations—has evidence to support its use". Instead students need to be taught to comprehend the situation described in the word problem.

"True comprehension occurs when one constructs a representation not only of the text itself, but also of the situation described by the text; both linguistic and mathematical knowledge." (Kitsch, 1986).

An Effective Approach

A more mathematically robust approach to solving word problems is to comprehend the words, numbers and graphics in the problem through the lens of problem structure/s.



Effective problem solving uses two scaffolds:

i. A general problem solving process such as the four steps devised by Polya (1954).

ii. A diagrammatic approach that represents the structure of the problem, such as a bar model.

i) Problem Solving Process

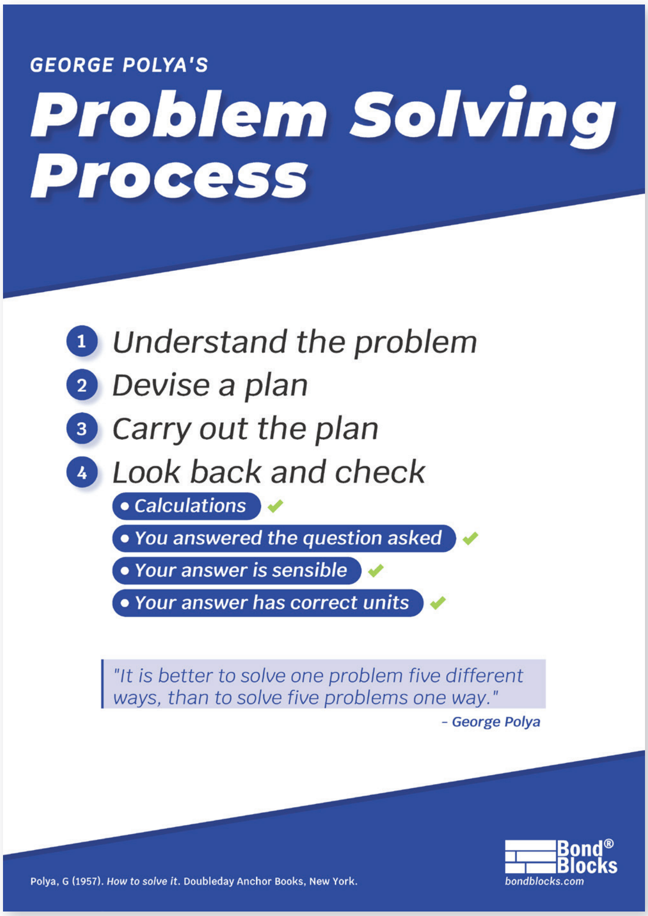
The four steps in Polya's problem solving process are:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

This problem solving approach is useful as a whole school approach for solving all different types of problems. Of which word problems are just one example. For very young children these steps may be shortened to **(1) Understand, (2) Plan, (3) Do, (4) Check**.

Polya's problem solving process is applied to Bond Blocks in more detail later in this document.

It is worth noting that **"Good problem solvers spend a bulk of time on Step 1 of Polya's method (understanding the problem and all relevant relationships) while novices rush to try a plan without really thinking through the plan's effectiveness."** Chapin, O'Connor & Anderson, 2009, p.97.



ii) Diagrammatic Approach

Using a diagram such as a bar model helps students do this. Bar model diagrams go by many names including strip diagrams (USA), tape diagrams (Japan), ribbon diagrams and Singapore Maths (introduced in the 1980s) [1]. Sawyer (1964) emphasised the importance of students drawing these diagrams to help them understand the mathematics. Bruner (1966) highlighted the importance of this in his stage of learning (Enactive, Iconic, Symbolic).

[1] *"The Ultimate Guide to Bar Modelling"*. Download from **www.thirdspacelearning.com**

Part-Part-Whole and Comparison Problems

Around the world there are multiple different ways of sorting additive (addition and subtraction) word problems into different structures (also referred to as "types", "schema based instruction" and "situations"). We have simplified these into two structures:

- Part-Part-Whole
- Comparison

All addition and subtraction word problems can be sorted into these two structures. It is important that students can identify each structure and the difference between them.

Typically students are exposed to part-part-whole problems more often than comparison problems, and find comparison problems more difficult than part-part-whole problems. For these reasons many students benefit from increased exposure to comparison problems.

Part-Part-Whole

Part-part-whole problems contain two parts that join together to make a whole.
Wholes can be partitioned into more than two parts.

<table>
<tr><td colspan="2">Whole</td></tr>
<tr><td>Part</td><td>Part</td></tr>
</table>

Part + Part = Whole

Whole - Part = Part

For example, Josh and his sister Grace combined their pocket money to buy a toy. Josh had $6. After Grace added her pocket money they had $10 altogether. *How much pocket money did Grace have?*

<table>
<tr><td colspan="2">Whole Known</td></tr>
<tr><td>Part Known</td><td>Part Unknown</td></tr>
</table>

<table>
<tr><td colspan="2">Whole
$10</td></tr>
<tr><td>Josh's Part
$6</td><td>Grace's Part
$4</td></tr>
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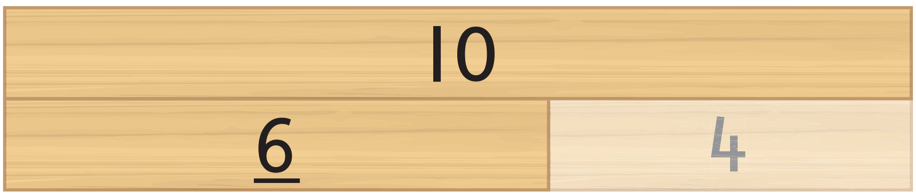


$$10 - 6 = \boxed{4}$$

Grace had **$4** pocket money.

Comparison Problems

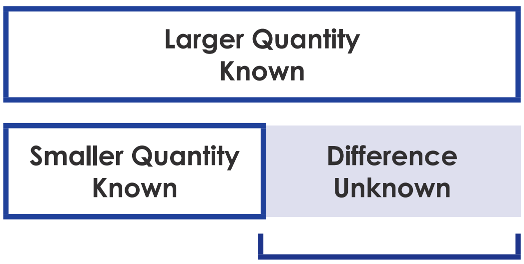
Comparison problems compare two quantities and use the difference between the two quantities. Comparison problems use comparative language such as "fewer/more", "less than/greater than".

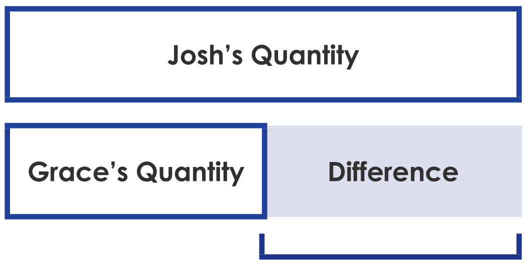
Smaller Quantity + Difference = Larger Quantity

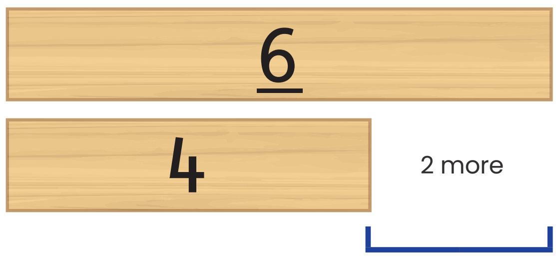
Larger Quantity - Smaller Quantity = Difference

Larger Quantity - Difference = Smaller Quantity

For example, Josh had $6. Grace had $4. *How much more money did Josh have compared to Grace?*







$$6 - 4 = \boxed{2}$$

Josh had **$2** more than Grace.

Active and Static Word Problems

Part-Part-Whole and Comparison word problems can be written in either an active or static way.

Active problems have an action where the parts (in a part-part-whole structure) or quantities (in a comparison structure) interact with each other. For example, in the following two questions the girls join the boys on the mat. This makes them active questions.

(Active / Part-Part-Whole)

There were 4 boys sitting on the mat. Six girls came and sat down to join them. How many children were sitting on the mat altogether?

Part-Part-Whole word questions written in an active way are sometimes referred to as Change or Join questions.

(Active / Comparison)

There were 4 boys sitting on the mat. Six girls came and sat down to join them. How many more girls were there compared to boys sitting on the mat?

In **static problems** the parts (in a part-part-whole structure) or quantities (in a comparison structure) do not interact with each other. For example, in the following two questions the girls and boys on sitting on the mat. This makes them static questions.

(Static / Part-Part-Whole)

There were 4 boys and 6 girls sitting on the mat. How many children were sitting on the mat altogether?

(Static / Comparison)

There were 4 boys and 6 girls sitting on the mat. How many more girls were there compared to boys sitting on the mat?

Classifying questions as active or static in unnecessary for students because this information is not needed to solve the problem. Whereas identifying the question as a part-part-whole or comparison structure is.

However, it is helpful for teachers to be aware of active and static ways of presenting word problems. Typically students are exposed to active problems more often than static problems. Active problems also tend to be easier for students to solve because they can be acted out at different levels (concretely, representationally or abstractly in your minds' eye). This makes comprehending the problem and identifying the needed operation easier. For these reasons many students benefit from increased exposure to static problems.

Missing Number Positions

The missing number, or 'unknown', can be placed in any position within the equation related to the word problem. This one example, "A fish tank contained 10 fish. Six were gold, four were black," can be used to write word problems where the missing number is placed in a **variety** of positions.

10	
6	4

Typically students are exposed to word problems where the missing number or 'unknown' is isolated on one side of the equation. For example, addition as *6 + 4 = ?* and subtraction as *10 - 6 = ?* These types of questions are shaded darker in the six examples below.

Finding the unknown in a position other than the answer position is often more difficult for students. These questions are shaded lighter in the six examples below. Students benefit from increased exposure to word problems where the missing number is in different positions.

Unknown	
6	4

10	
Unknown	4

10	
6	Unknown

Static Examples

$6 + 4 = \boxed{?}$

There were 6 gold fish and 4 black fish in a tank. The tank had no other fish.

How many fish were in the tank?

$\boxed{?} + 4 = 10$

There were some gold fish in a tank and 4 black fish. Altogether there were 10 fish.

How many were gold?

$6 + \boxed{?} = 10$

There were 6 gold fish in a tank and some black fish. Altogether there were 10 fish.

How many were black?

Active Examples

$\boxed{?} - 6 = 4$

There were some fish in a tank. Six gold ones were moved to a pond. This left 4 black fish in the tank.

How many fish were in the tank to start with?

$10 - \boxed{?} = 4$

There were 10 fish in a tank. Some were gold. The gold fish were moved to a pond. This left 4 black fish in the tank.

How many fish were gold?

$10 - 6 = \boxed{?}$

There were 10 fish in a tank. Six gold ones were moved to a pond. The fish left in the tank were all black.

How many were black?

Supporting Students Who Have Difficulty

Identify the stage of difficulty

Newman (1983) identified five stages students progress through when solving word problems. Different errors arise at each stage and are identified by interview cues.

Newman Error Analysis Stage	Newman's Interview Cue	Related Stage in Polya's Problem Solving Process
Reading and Decoding The student reads the problem and decodes words and symbols.	Please read the question to me. If you don't know a word, leave it out.	**Understand the problem**
Comprehending The student make sense of what they have read.	Tell me what the question is asking you to do.	
Transforming The student 'mathematises' the problem; that is, works out what maths is needs to be done.	Tell me how you are going to find the answer.	**Devise a plan**
Processing The student does the maths.	Show me what to do to get the answer. 'Talk aloud' as you do it, so that I can understand how you are thinking.	**Carry out the plan**
Encoding The student records their final result appropriately.	Now, write down your answer to the question.	**Look back and check**

Numberless problems

Exposure to “numberless problems” (Bushart, 2014), prior to solving word problems with numbers, can significantly increase understanding.

There are two distinct steps we need to take when solving a word problem. First we must work out the mathematical relationship between the quantities and, second, we must operate on the quantities. The first step is totally independent of the actual size of the quantities. (Askew, 2019, p. 56)

By engaging with word problems that are numberless students focus on understanding the relationships between the numbers. Numbers are introduced in gradual steps so as the students “talk about the relations between quantities without knowing the actual quantities. As specific numbers are gradually introduced so the reasoning about what to do with those numbers emerges” (Askew, 2019, p 56).

For example, this word problem could be scaffolded, beginning with a numberless word problem, using the following steps:

? + 4 = 10

There were some gold fish in a tank and 4 black fish. Altogether there were 10 fish.

How many were gold?

Steps:

1. There were fish in a tank. Some were gold. The other fish were black.
2. There were 10 fish in a tank. Some were gold. The other fish were black.
   (This is an ideal place to pause and investigate possibilities.)
3. There were 10 fish in a tank. Some were gold. The other 4 were black.
   How many fish were gold?

References

Askew, M. (2019). *Help students understand the structure of maths problems before introducing numbers.* Teachwire. Advance online publication. https://www.teachwire.net/news/help-students-understand-the-structure-of-maths-problems-before-introducing-numbers

Bruner, J. S. (1966). *Toward a theory of instruction*. Cambridge: Harvard University Press.

Bushart, B. (n.d.). *Numberless Word Problems.* Teaching to the beat of a different drummer. https://bstockus.wordpress.com/numberless-word-problems

Newman, A. (1983). *The Newman language of mathematics kit: Strategies for diagnosis and remediation*. Sydney: Harcourt Brace Jovanovich Group.

Polya, G. (1954). *How to solve it*. Princeton, NJ: Princeton University Press.

White, A. L. (1999). *A revaluation of Newman's error analysis. In Mathematics of Prime Importance.* Proceedings of The Mathematical Association of Victoria Annual Conference.

Willis, S., Devlin, W., Jacob, L., Powell, B., Tomazos, D., & Treacy, K. (2004). *First Steps in Mathematics Number: Book 2*. WA: Department of Education.