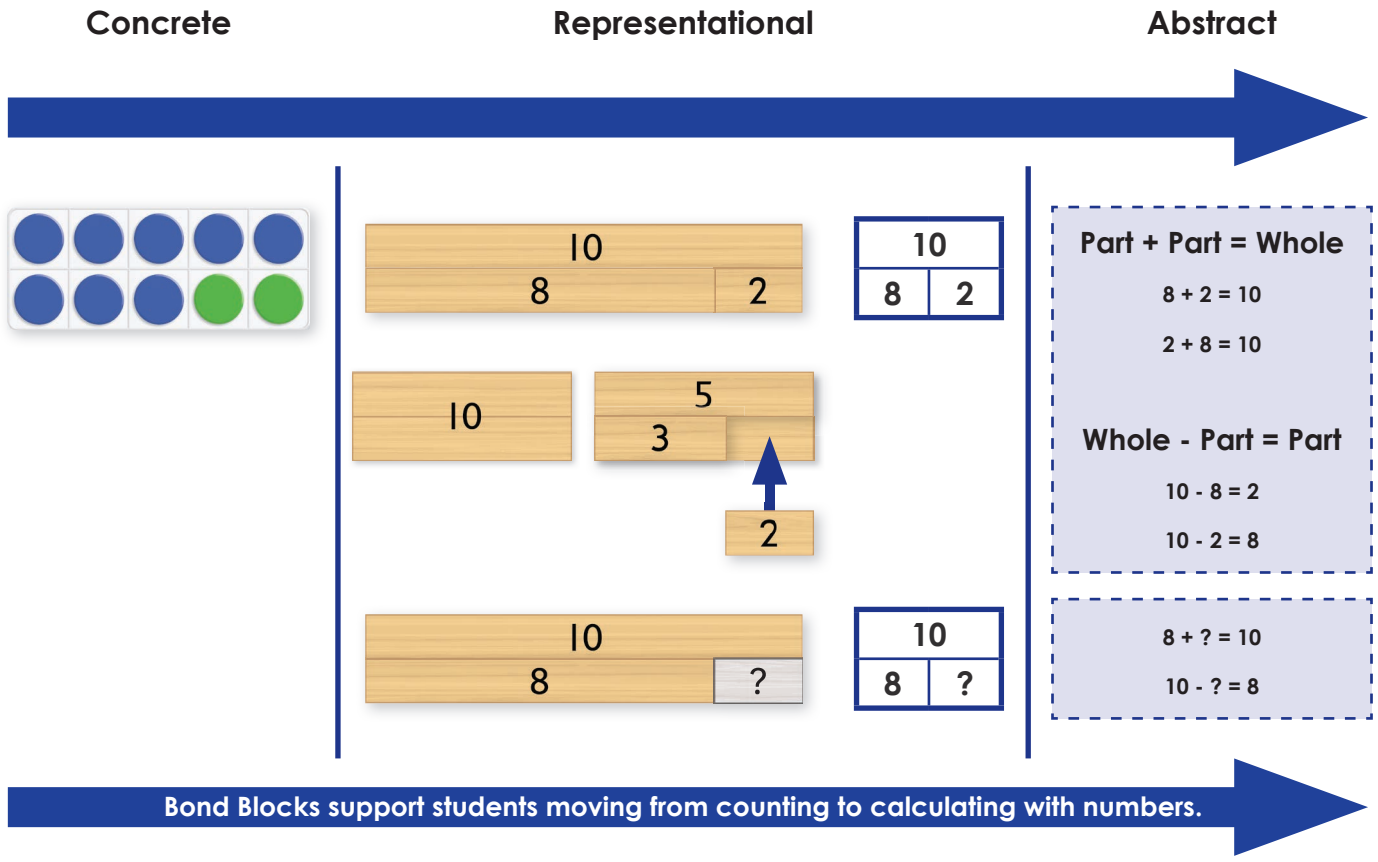
Developing Calculation, Not Counting

Developing Counting Principles

Initially Bond Blocks can be used in conjunction with discrete countable objects such as counters whilst students develop the principles of counting (Gelman & Gallistel, 1978).

1. The stable order principle:
   - Number names are said in the conventional order.
2. The one-one principle:
   - Each item is counted once as the corresponding word is said.
3. The cardinal principle:
   - The last number said indicates the total for the group.
4. The order-irrelevance principle:
   - Counting left to right, right to left or in a scattered arrangement does not change the quantity.
5. The abstraction principle:
   - Physical items that differ in type, colour and size can be counted.
   - Things that cannot be touched such as sounds and ideas can be counted.

These principles need to be grasped before the counters are removed and only Bond Blocks are used. This follows Bruner's (1966) approach of moving from:



Counting or Calculating

The Bond Blocks system makes the distinction between counting and calculating clear. The Bond Blocks Core Kit is used to help students move from counting to adding and subtracting (without counting).

When first learning to add and initial strategy students first learn is counting on from the larger number (zero, one, two or three). Similarly, when beginning to subtract, students initially use the strategy of counting back (zero, one, two or three). This strategy needs to be replaced by more efficient strategies. For this reason, it is recommended that when teaching students should not count on or back more than three.

If students do count on or back more than three to solve addition or subtraction the teacher can:

i. Provide different manipulatives.
ii. Model a different strategy.
iii. Change the size of the numbers in the question to be less.

Doing these things will help students learn and practice efficient calculating strategies. It will also help prevent students from becoming entrenched in using counting as their primary strategy to solving addition and subtraction.

Students who hold onto counting to add and subtract **are at mathematical risk**. Examples of counting by ones to calculate include tapping fingers, eye nods, drawing lines on paper, using the marks on a ruler or making repeated jumps of one on a number line.

Moving from Counting to Calculating

Concrete

Most manipulatives can be counted by ones.

Many students don't progress from counting by ones.

The Missing Link

Representational

Bond Blocks are a physical manipulative to support Singapore, bar-model maths.

The Missing Link to Abstract Calculation

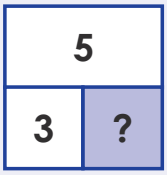
Bond Blocks cannot be counted by ones.

- Learn number bonds (facts) in a self-correcting way.
- Relate addition and subtraction using Part-Part-Whole.

Abstract

3 + 2 = 5
2 + 3 = 5
5 - 3 = 2
5 - 2 = 3

Extend with algebraic thinking.



5 - ? = 3
3 + ? = 5

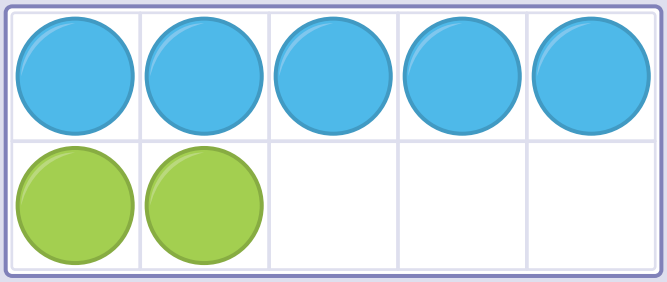
Bond Blocks fill the **missing link** to help students move from concretely **counting by ones** to abstractly **adding with numbers and symbols**.

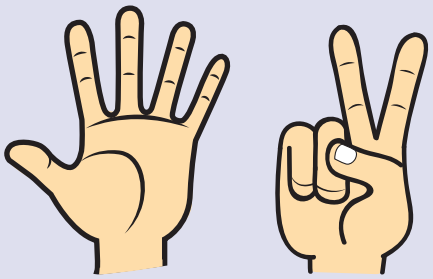
Structuring Materials to Develop Calculation

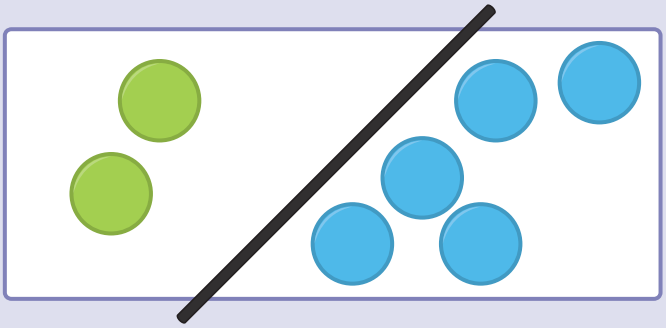
There are many structured materials that will help students help to calculate. These include:

- Using ten frames to scaffold 5 plus bonds, doubles and near doubles.
- Using partition boards to separate counters into easily seen groups.
- Organising counters into standard arrangements, such as those found on playing cards and dominoes, so students identify a quantity without counting all.
- Using fingers in a subitised way instead of counting each finger by ones.

7 as a 5 plus bond

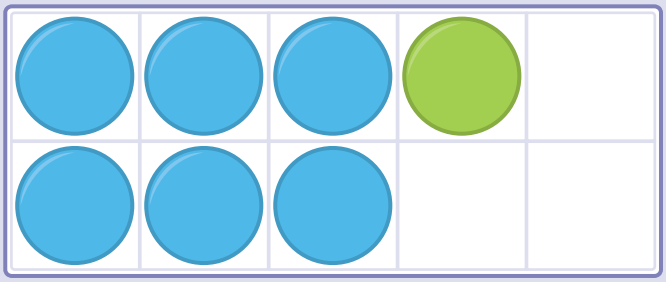


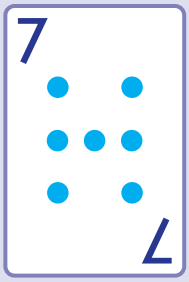


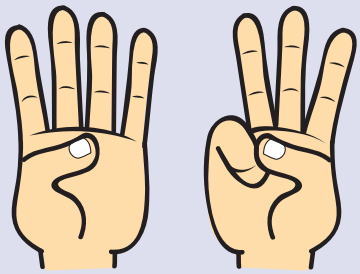


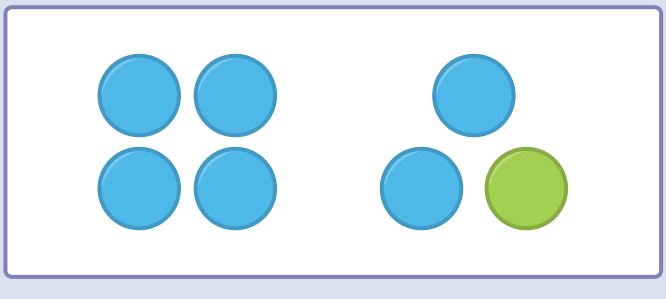
Partition board

7 using near doubles









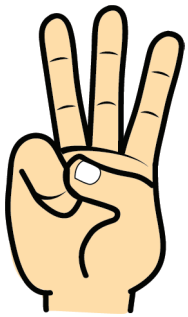
Standard arrangements

Using counters in a non-structured way is not wrong but it teaches counting, not calculating. The Bond Block system makes the distinction between counting and calculating clear.

Subitised Use of Fingers

"If finger counting by ones to add and subtract is such a problem why then do fingers appear in the Bond Block Core Kit activities?"

The previous section described how using fingers to add and subtract by counting in ones can be detrimental. This section describes how fingers can be used productively, in a subitised way to develop calculation. Subistising means seeing without counting. This is different to using fingers to track counting one by one.



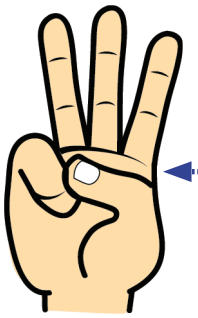
I can make three by putting up all three fingers at once (not counting them up one at a time).

Using fingers in a subitised way can be the concrete step in a concrete-representational-abstract approach. The next step is to move on from fingers to the representational stage of using Bond Blocks. Finally, we want them to stop using the blocks and move onto the abstract stage of using numbers and symbols.

I can see part-part-whole.

I can see and feel "3 and 2 is 5".

3 + 2 = 5



Part (fingers up) say, "3".

Part (fingers down) say, "and 2".

Whole (hand) say, "is 5".

So this means "2 and 3 is also 5".

2 + 3 = 5

I can use this to work out.

5 - 3 = 2

5 - 2 = 3

The research refers to this as "finger gnosis". For more information see:

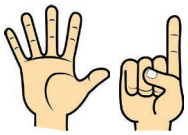
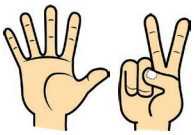
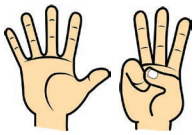
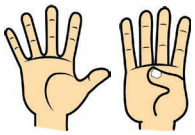
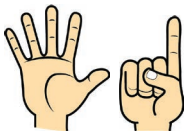
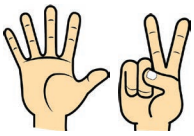
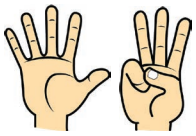
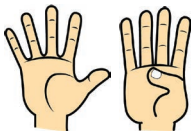
Bisnaz, J., Fast, L., Kamawar, D., LeFevre, J., Penner-Wilger, M., Skwarchuk, S., & Smith-Chant, B. (2009). Subitizing, Finger Gnosis, and the Representation of Number. Proceedings of the Annual Meeting of the Cognitive Science Society, 31.

Systematic Subitising

Using fingers in a subitised way to add and subtract is taught systematically through the Bond Blocks Core Kit in two different ways:

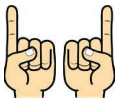
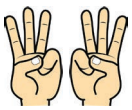
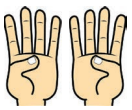
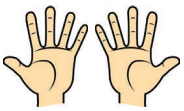
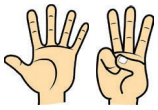
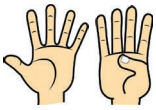
- Bonds related to five. This includes **'Bonds of 5'**, **'Five Plus Bonds'**, **'Bonds of 10'**.
- Bonds related to doubles. This is **'Doubling and Halving to 10'**, then **'Doubling and Halving to 20'**.

Bonds Related to Five

Chapter 2 – Bonds of 5 Once students can make Bonds of 5 they use these to make Five Plus Bonds.	 5 as 1 and 4	 5 as 2 and 3	 5 as 1 and 4	 5 as 1 and 4
Chapter 4 – Five Plus Bonds Once students can make Five Plus Bonds they use these as Bonds of 10.	 6 as 5 and 1	 7 as 5 and 2	 8 as 5 and 3	 9 as 5 and 4
Chapter 5 – Bonds of Ten Fingers up represent one part. Fingers down represent the other part. Both hands represent the whole.	 10 as 6 and 4	 10 as 7 and 3	 10 as 8 and 2	 10 as 9 and 1

Bonds Related to Doubling and Halving

Fingers are used in a different arrangement for doubles and near doubles.

Doubles and Halves Each hand shows the same number of fingers.	 2 as 1 and 1	 4 as 2 and 3	 6 as 3 and 3	 8 as 4 and 4	 10 as 5 and 5
Near Doubles One hand has one more or less than the other.	 3 as 2 and 1	 5 as 3 and 2	 7 as 5 and 3	 9 as 5 and 4	

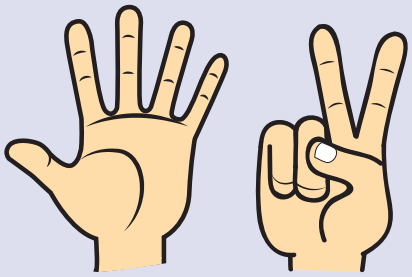
Using Fingers to Develop Calculation

Students need to trust the quantity of the whole and the partitions into which the whole is split. To move students away from finger counting by ones, students are encouraged to trust the quantity by using their fingers to subitise the whole. Subitising means seeing the quantity WITHOUT counting. This is very different from using fingers to count.

Using fingers in a subitised way reinforces;

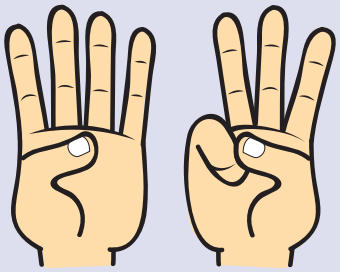
- The number bond and partitioning of the whole. For example, 7 partitioned into the bond of 5 and 2.
- Both concepts of addition and subtraction, as well as relationships between them.
- Solving subtraction by either taking away or adding on.

7 as 5 and 2

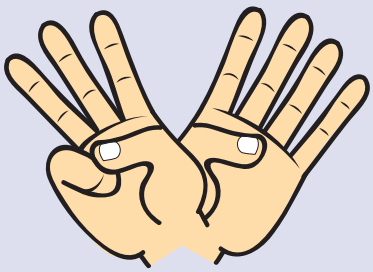


5 + 2 = 7
2 + 5 = 7
7 - 5 = 2
7 - 2 = 5

7 as 4 and 3



3 + 4 = 7
4 + 3 = 7
7 - 3 = 4
7 - 4 = 3



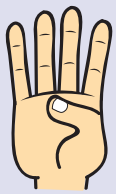
Swap hands to show the commutative property of addition.

Finger Subitising to Support 5 Plus Bonds

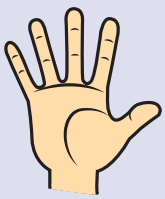
1

2

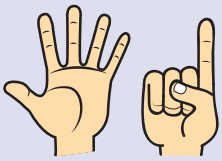
3



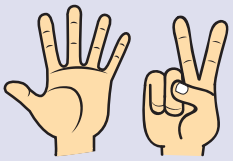
4



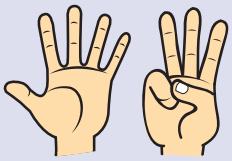
5



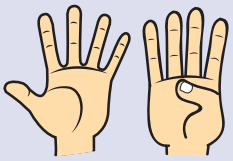
6 as 5 + 1
so
1 + 5 = 6
6 - 5 = 1
6 - 1 = 5



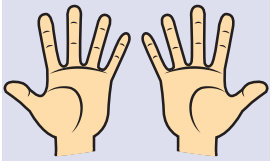
7 as 5 + 2
so
2 + 5 = 7
7 - 5 = 2
7 - 2 = 5



8 as 5 + 3
so
3 + 5 = 8
8 - 5 = 3
8 - 3 = 5



9 as 5 + 4
so
4 + 5 = 9
9 - 5 = 4
9 - 4 = 5



10 as 5 + 5
so
10 - 5 = 5

Shown with right hand dominance. The fingers on the right hand change each time.

Note: Students with fine motor difficulties may not be able to separate fingers easily or use their thumb to hold other fingers. Do not insist these students use a representation that is difficult for them. Consult their occupational therapist for advice.

Students At Mathematical Risk

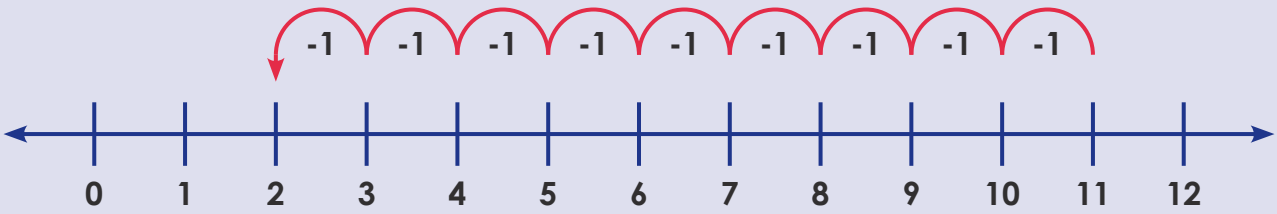
Research repeatedly highlights that students at mathematical risk or those who have a specific learning difficulty in mathematics (Dyscalculia):

1. **Lack of fluent recall of number bonds.**
2. **Lack of an understanding of the properties of addition and subtraction and relationships between them including:**
   - The commutative property of addition
   - Part-part-whole
   - The inverse relationship between addition and subtraction
3. **Lack flexible, efficient calculating strategies which use these understandings.**
4. **Persist in counting by ones to add and subtract.**

Students at mathematical risk or those who have a specific learning difficulty in mathematics (Dyscalculia) typically solve subtraction by taking away only. For example, to solve 11-9, they start at 11 and count back 9 by one.

Subtraction solved using Taking Away

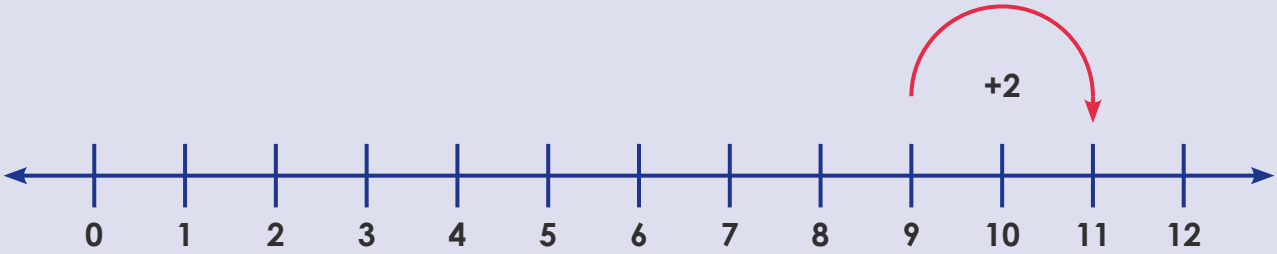
- **Inefficient counting by one**
- **Error prone**
- **Leads to dependence on counting**



This is an inefficient, incomplete understanding of subtraction that lacks the foundation for more abstract mathematics including algebra. Whereas 11-9 is more efficiently solved using 9+2.

Subtraction solved using Adding On

- **Efficient partitioning**
- **Leads to development of increasingly complex mathematics.**

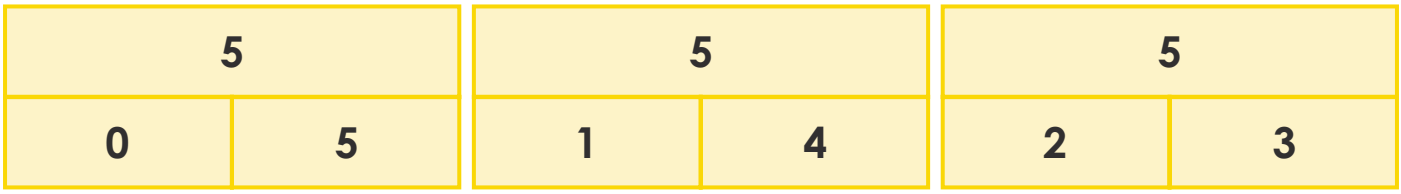


Students who use counting forwards and backwards by ones, as their principle strategy to add and subtract, are at significant risk of ongoing difficulties in mathematics. Examples of counting by ones to calculate include tapping fingers, eye nods, drawing lines on paper, using the marks on a ruler or making repeated jumps of one on a number line.

Counting forwards or backwards more than three impedes the development of calculation for these reasons:

- Students who use counting as their primary strategy to solve addition and subtraction become entrenched in trusting this strategy. Because they choose counting, they do not practise new strategies enough to develop fluency.
- Effective mathematicians select efficient calculating strategies based on the size of the numbers and their situation. Counting more than three is error prone and inefficient.
- Using fingers to count add or subtract involves a double count. The student has to count forwards or backwards whilst tracking how many to add or take away. Working memory is stretched when using inefficient methods such as those involving "double counting".

Visuals support students with memory difficulties to engage in higher order mathematical thinking whilst increasing student trust and use of number bonds.



Bonds of 5 Desk Visual

Bond Blocks were designed to combat these difficulties and move students from counting to calculating using addition and subtraction.

References

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