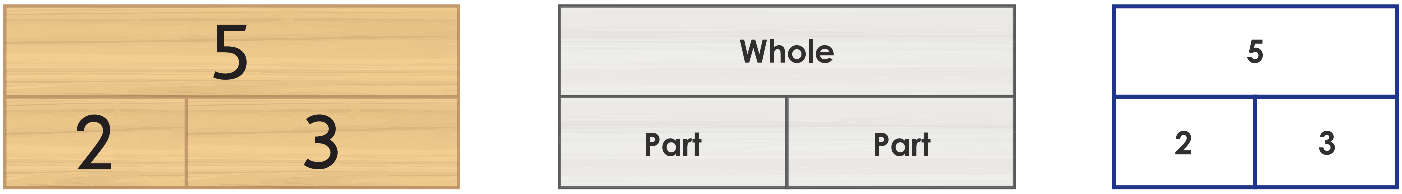
Bond® Blocks TEACHER NOTES

Building Equations: Using Part-Part-Whole

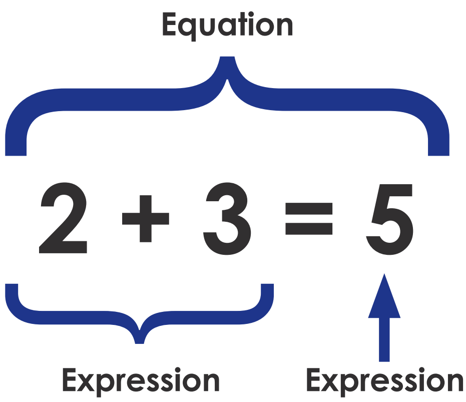
The concept of part-part-whole is used to teach addition and subtraction.

The part-part-whole diagram is a diagrammatic representation of the number bond. It is different to Bond Blocks in that it is not to scale.



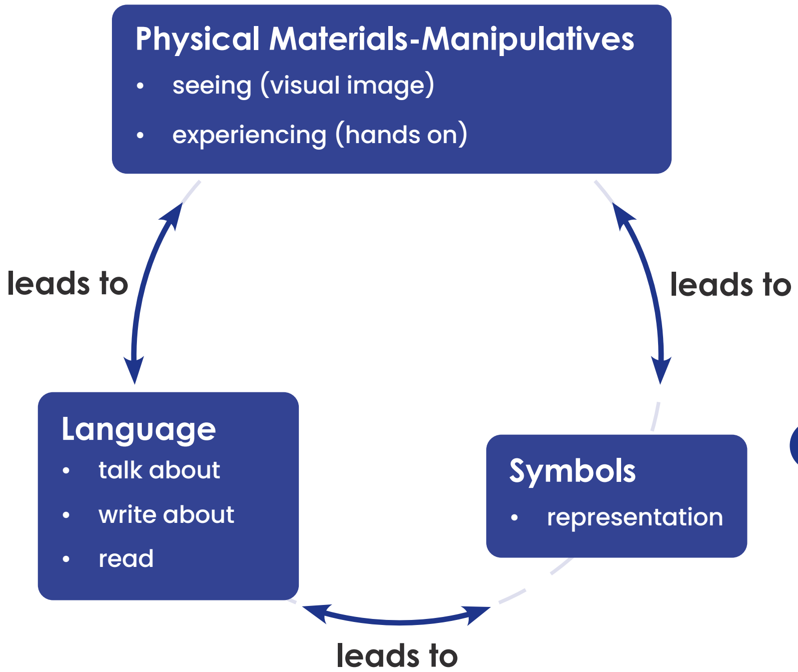
In order to develop a robust understanding of addition and subtraction the concept of “equals” need to be understood.

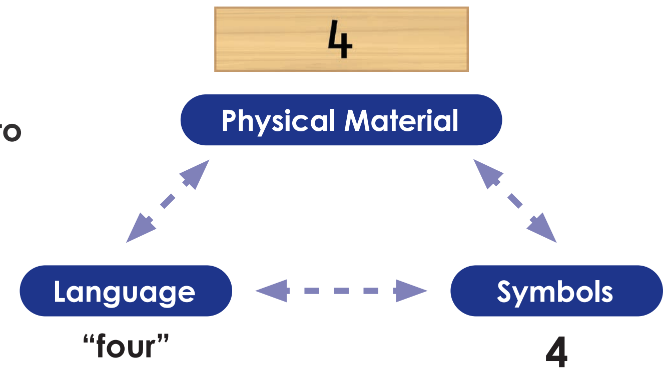
- **Equals** means “is”.
- An **equation** contains an **equals symbol**. =
- Either side of an equation is an **expression**.



For example, in the equation 2 + 3 = 5 the expression on the left side of the equals symbol is 2 + 3 and the expression on the right side of the equals symbol is 5.

The following section outlines two different ways to represent equals using manipulatives. In each representation explicit links have been made between the manipulative, language and symbols.





Equals using One Set of Manipulatives

Frequently in primary schools "equals" is taught using one set of manipulatives.

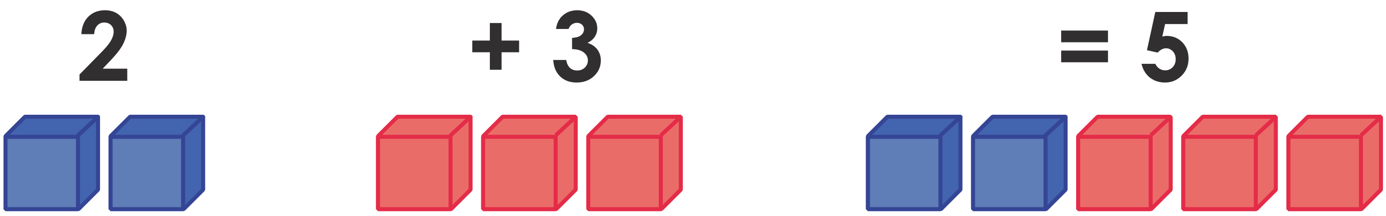
- When adding, one amount of manipulatives is collected (one part), another amount is added (the other part), then the manipulatives are combined to make one set (the whole).

  Part + Part = Whole

- When subtracting, one set of manipulatives is collected (the whole), an amount of this set is separated (one part), leaving the remaining manipulatives (the other part).

  Whole – Part = Part

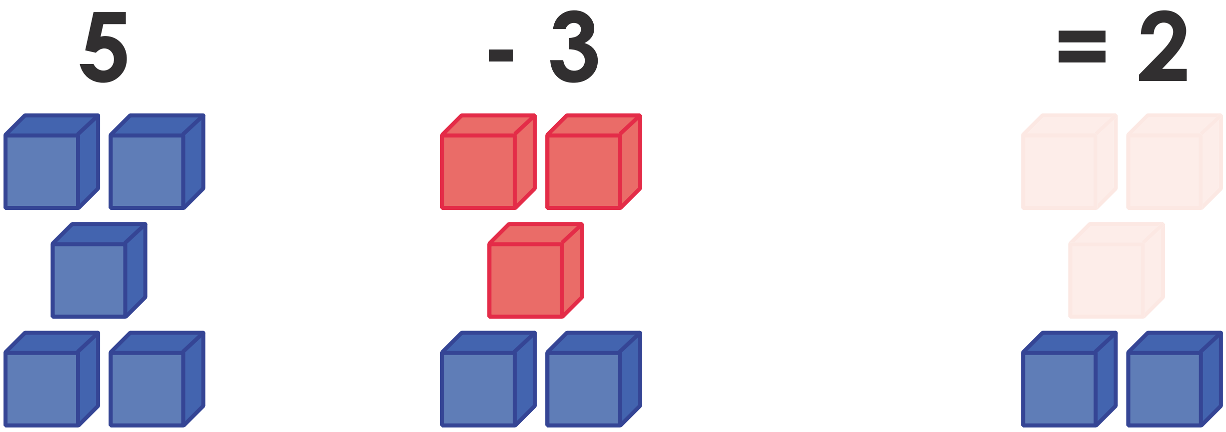
This example shows **2 + 3 = 5** which can be read as **"Two add three is five"**. It uses one set of 5 blocks.



First collect two blocks. **Then add three more.** **There are five altogether.**

This example shows **5 - 3 = 2** which can be read as **"Five subtract three is two"**. It uses one set of 5 blocks.

If students are only exposed to using equals in this type of representation they can develop:



Start with 5 blocks. **Then subtract 3.** **This leaves 2.**

- Misconceptions. For example, thinking *5 = 2 + 3* is written 'back to front'.
- Incomplete understandings. For example, only thinking about equals as an active "makes" which causes difficulties when solving static or comparison word problems.

Equals Balancing Two Sets of Manipulatives

Equals can also be represented as balancing two sides of an equation. This involves using two sets of manipulatives, one set on each side of the equation, and adjusting the sets balance with each other. When this concept of equals has been used in Bond Blocks the phrase "is equal to" has been used.

The understanding of equals as balancing two sets is used to:

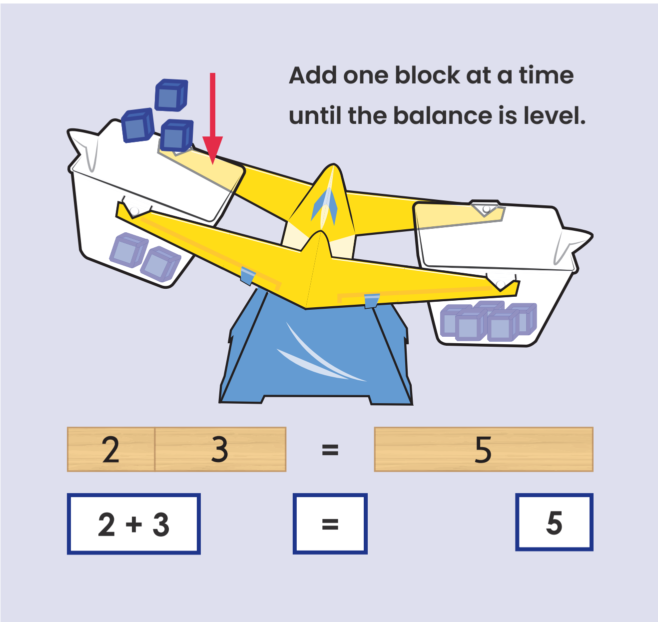
- Write equations in a non-standard way such as *2 = 5 - 3* or *3 + 2 = 2 + 3.*
- Solve comparison addition and subtraction word problems.
- Build a robust understanding of part-part-whole thinking.
- Lay the foundation for algebra.

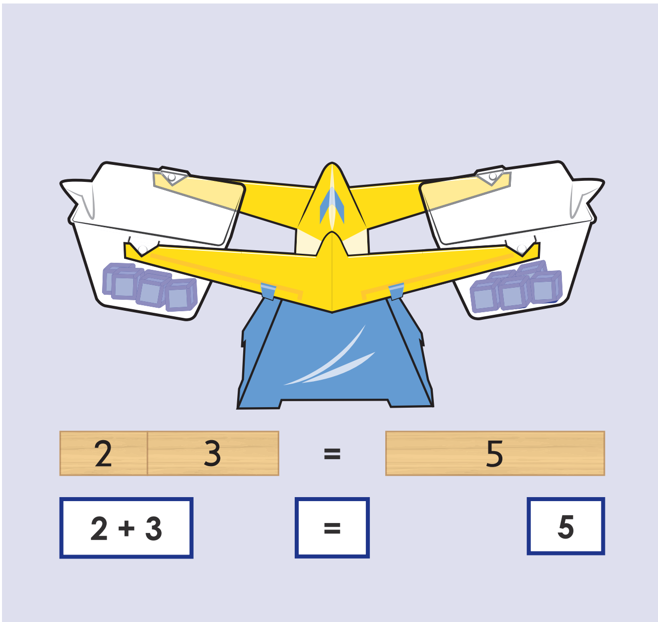
The Part-Part-Whole diagram, when used with to balance equations with manipulatives, uses two sets of manipulatives, one set on each side of the equation.

Whole	**This set of manipulatives**	5
=	**" is equal to "**	=
Part | Part	**this set of manipulatives.**	3 | 2

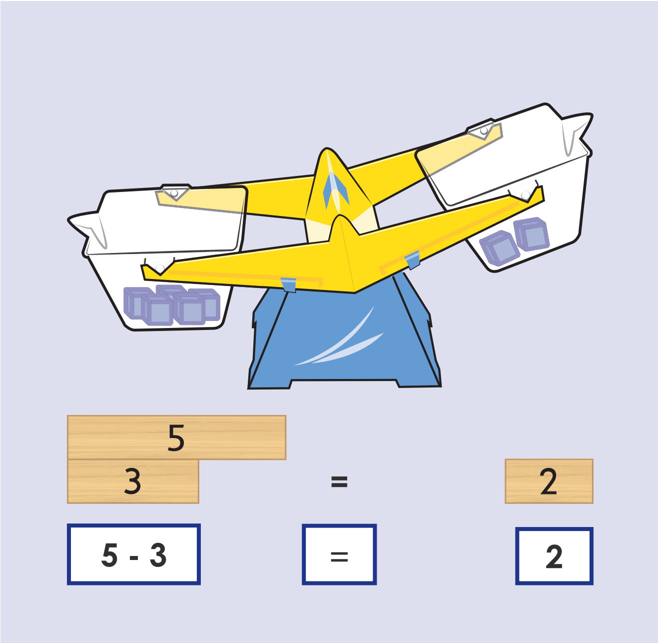
For activities involving balancing you will need to use plastic blocks because they are equally weighted. Bond Blocks are a linear system. **They will not balance because they are made of natural wood.**

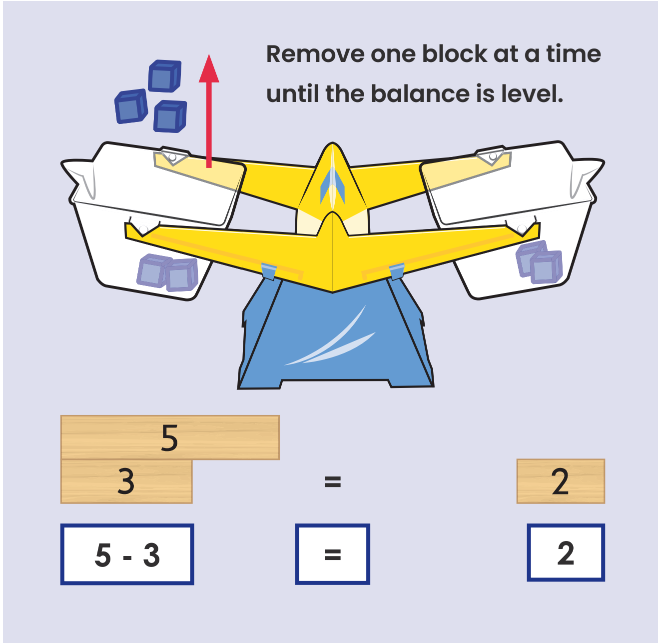
This example shows *2 + 3 = 5.* It uses two sets of 5 blocks.





This example shows *5 - 3 = 2.* It uses two sets of blocks.





Using the balance is an effective way to show the inverse between addition and subtraction, as indicated by the red arrows showing the adding or removing of blocks. The balance is a powerful visual when teaching inequality using greater than > or less than <.

Concrete Representational Abstract

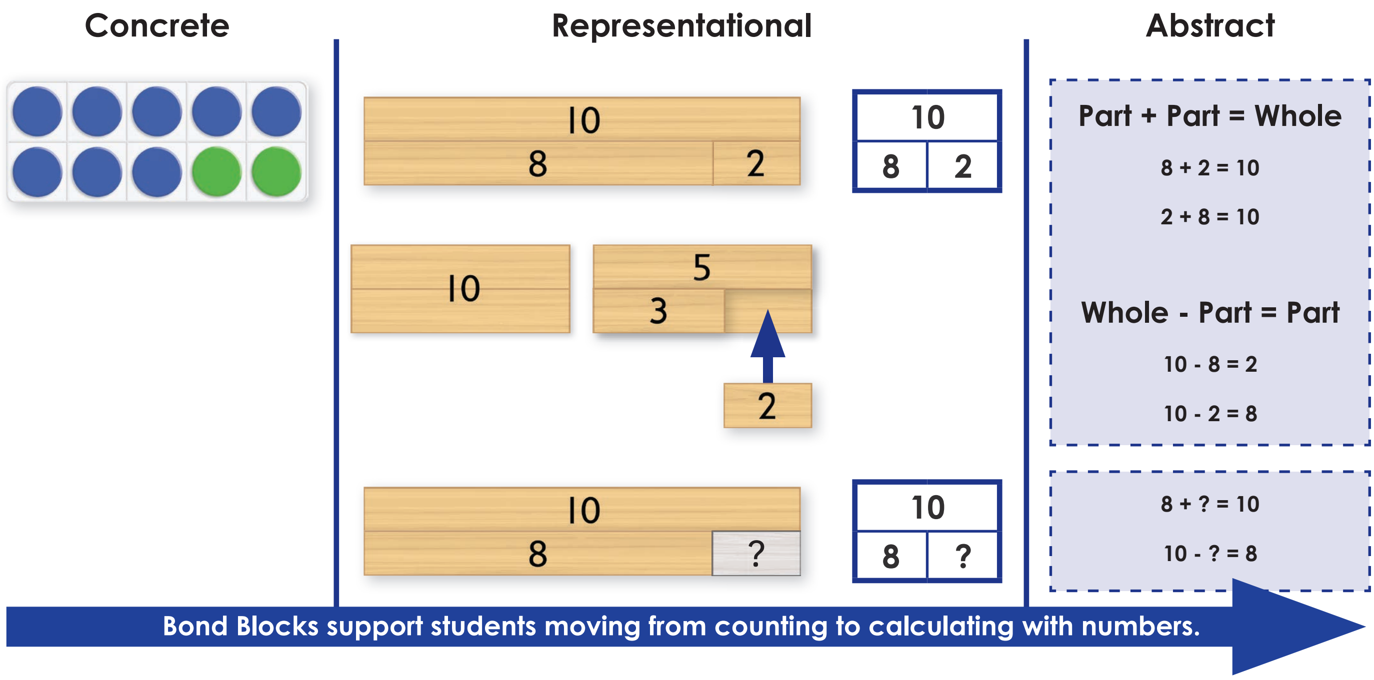
Bond Blocks fit into the teaching sequence after concrete manipulatives and before abstract written calculation. When teaching mathematics, students progress:

i) From a concrete understanding of equals as balance using manipulatives with one-to-once correspondence on a physical balance,

ii) To a representative understanding of balance using Bond Blocks,

iii) To an abstract understanding of balancing equations with numbers and symbols.

Students need to be exposed to multiple representations of equals to develop a robust understanding about equations.



Addition and Subtraction Using Part-Part-Whole

Students will need to be taught that for:

Addition

Part + Part = Whole

Subtraction

Whole - Part = Part

When adding the parts may be rearranged. Addition is commutative. Subtraction is not commutative.

Whole	
Part	Part

7	
4	3

4 + 3 = 7

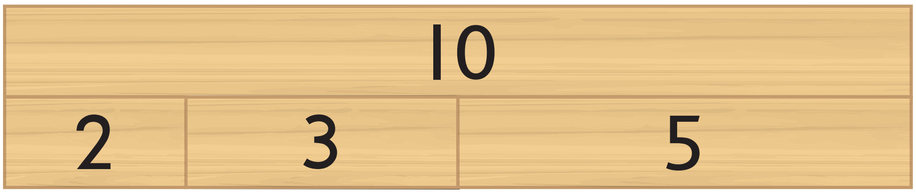
3 + 4 = 7

7 - 4 = 3

7 - 3 = 4

Extending Part-Part-Whole

Wholes can be made up of more than two parts. Here the whole of 10 has been partitioned into three parts.



Whole		
Part	Part	Part

10		
2	3	5

Activities may be made a little harder by using bonds of more than two parts.

Addition

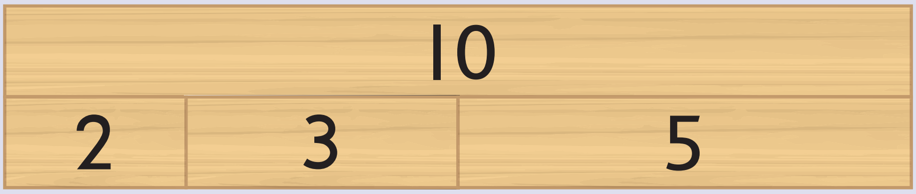
Part + Part + Part = Whole

Subtraction

Whole - Part - Part = Part

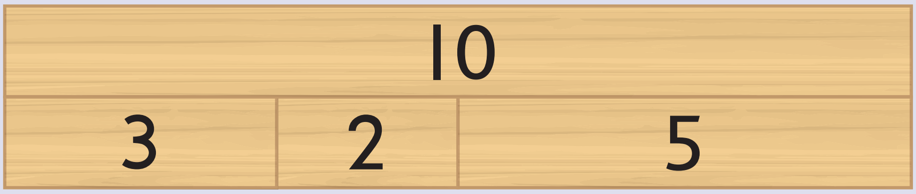
Parts can be rearranged because addition is commutative.

2 + 3 + 5 = 10



2 + 5 + 3 = 10

3 + 2 + 5 = 10



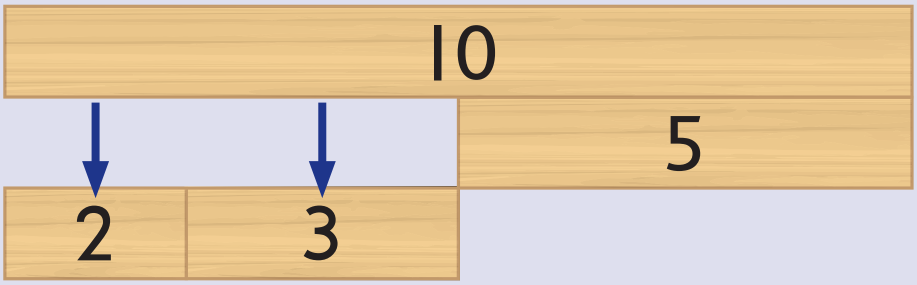
3 + 5 + 2 = 10

5 + 2 + 3 = 10

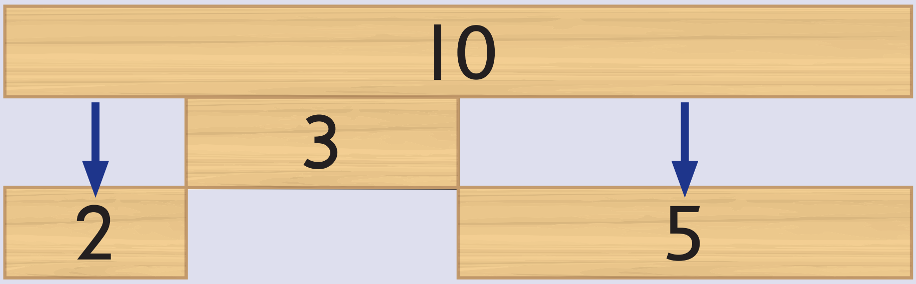
5 + 3 + 2 = 10

Parts have been subtracted from this part-part-whole representations.

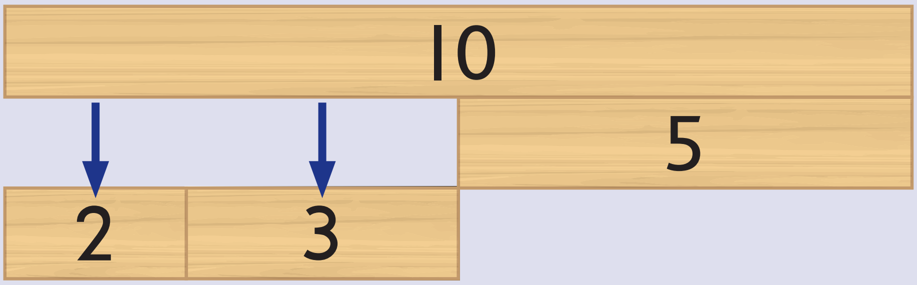
10 - 2 - 3 = 5



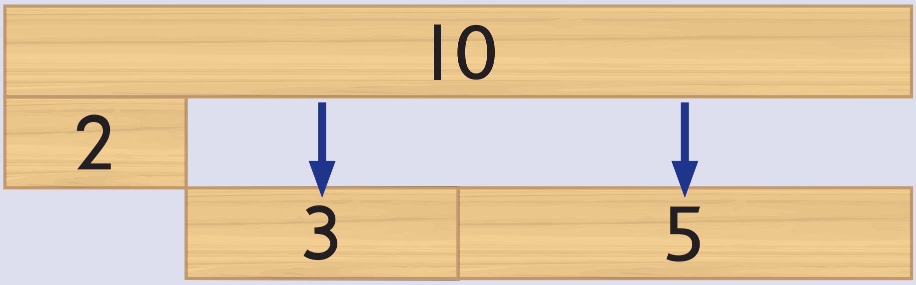
10 - 2 - 5 = 3



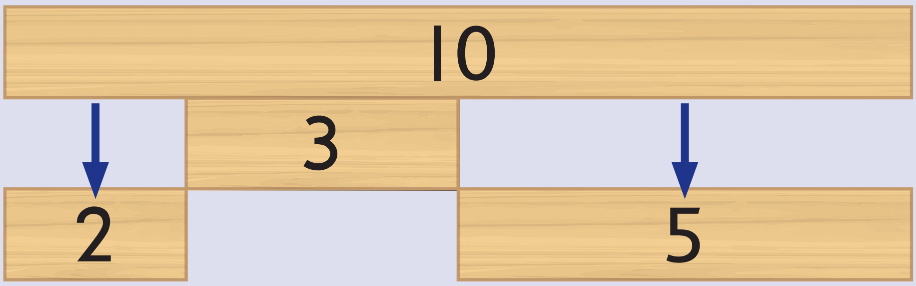
10 - 3 - 2 = 5



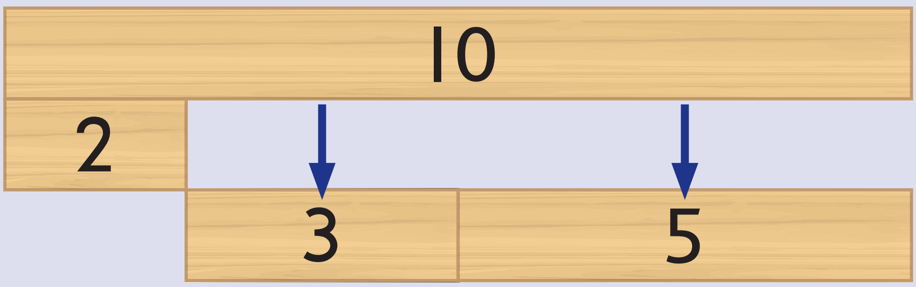
10 - 3 - 5 = 2



10 - 5 - 2 = 3



10 - 5 - 3 = 2



Atypical Arrangement using Part-Part-Whole

Addition and subtraction equations don't always have to be written as:

Part + Part = Whole

Whole - Part = Part

An alternative arrangement is:

Whole = Part + Part

Whole	=	Part	+	Part

Part = Whole - Part

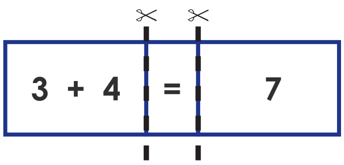
Part	=	Whole	-	Part

To teach this you will need:

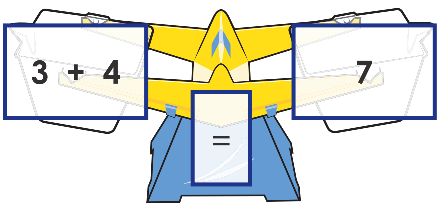
- one bucket balance.
- small ball of sticky tack to attach the card to the bucket balance.
- one piece of blank card approximately the same size as a bucket of the balance being used.
- a thick marker to write on the blank card.
- a pair of scissors.

Instructions

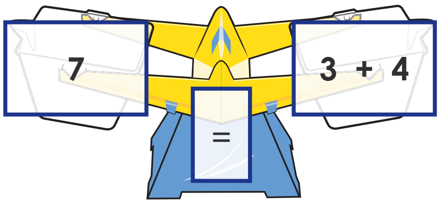
- Write an addition equation, in the typical order (part + part = whole), on the card.
- Cut the equation either side of the equal sign.



- Stick these to the buckets and fulcrum of the balance in typical order. Read from left to right.

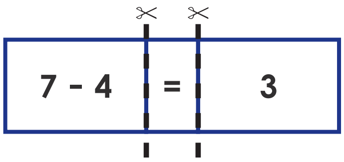


- Pick up the buckets and swap sides. This will produce the equation in atypical order. Reinforce that each side of the equals sign is read from left to right.

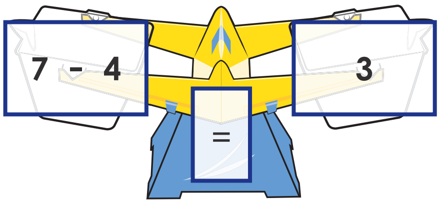


Repeat for subtraction

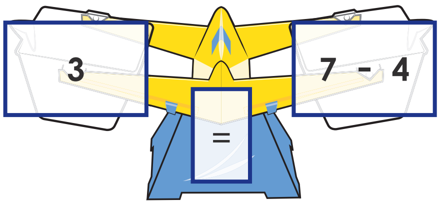
- Write an subtraction equation, in the typical order (whole – part = part), on the card.
- Cut the equation either side of the equal sign.



- Stick these to the buckets and fulcrum of the balance in typical order. Read from left to right.



- Pick up the buckets and swap sides. This will produce the equation in atypical order. Reinforce that each side of the equals sign is read from left to right.



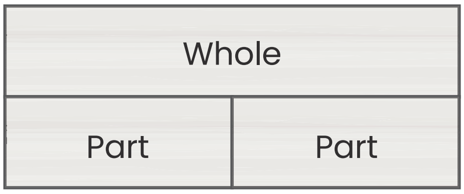
This method prevents students making the common error of writing the subtraction equation incorrectly when attempting atypical order.

Common Error.
Incorrect atypical arrangement.

$7 - 4 = 3$	$3 \neq 4 - 7$

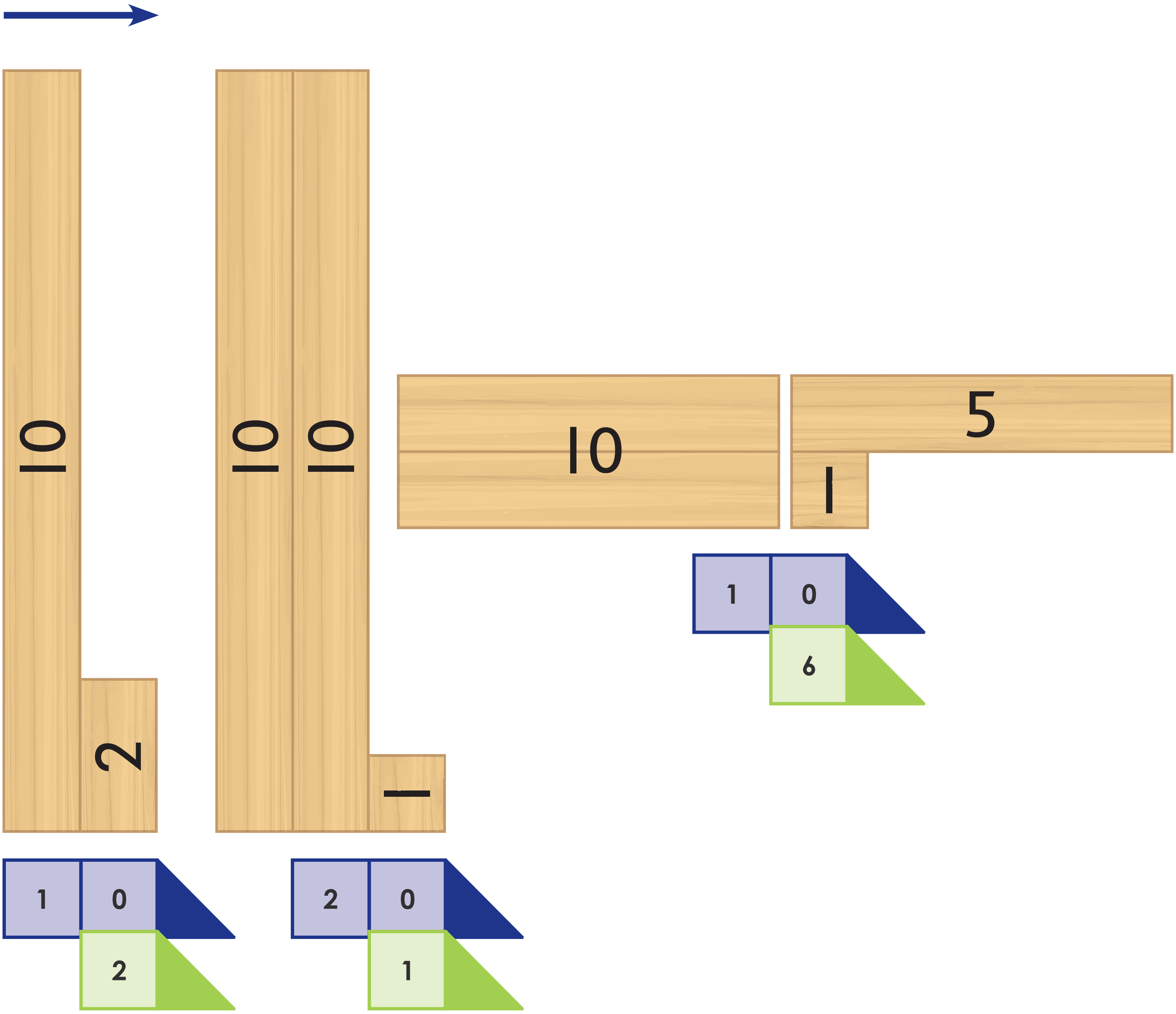
Block Placement

When using blocks to represent the whole and parts, place the whole on top and the parts underneath as per the part-part-whole diagram representation.



When placing parts generally the larger part is placed first, as long as this order is supported by the calculating strategy being used. Prior knowledge of the commutative property is needed to do this.

Work from left to right when placing blocks to support place value.

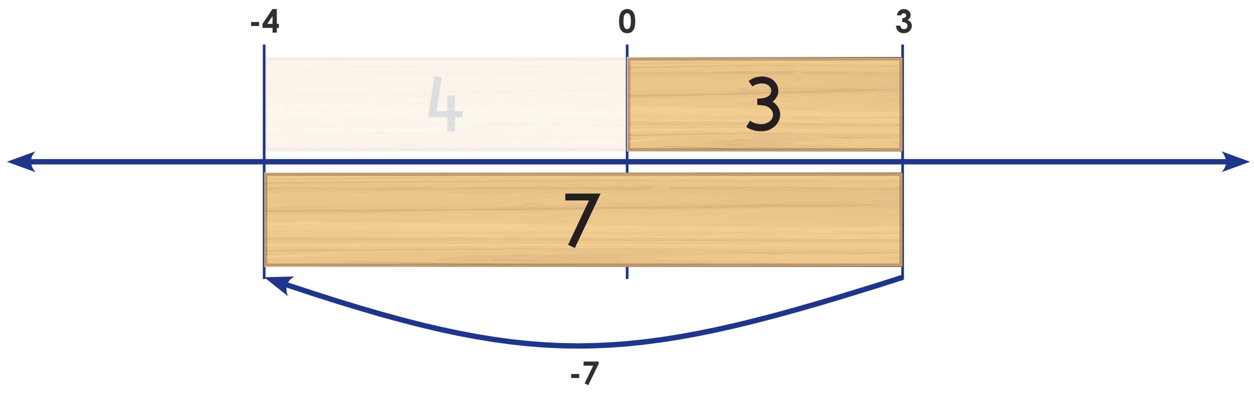


Once students demonstrate understanding and fluency they do NOT need to use the blocks.

A Note On Negatives

Often when students are creating subtraction equations they will incorrectly rearrange the whole and part. Students often write *3 - 7 = 4 which is incorrect*, when they mean *7 - 3 = 4* .

This can be demonstrated with Bond Blocks on an empty number line

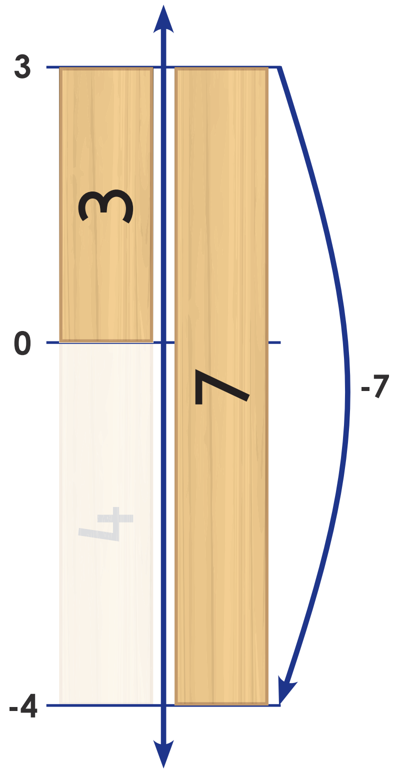


3 - 7 = -4 **3 - 7 ≠ 4**

For example, to demonstrate 3 – 7, start with 3 as the whole, on top of the empty number line. Subtract 7 by taking it away, placing it below the number line. The result is less than zero, four less than zero.

Some students find this easier to visualise if the number line is orientated vertically, as a thermometer.

eg. If it is 3 degrees and the temperature drops 7 degrees the temperature will be below zero.



When students write 3–7, but mean 7–3, teachers commonly try to correct this error by saying,

✗
- “Show me 3. You cannot take away 7” or
- “When taking away you need to put the bigger number first”.

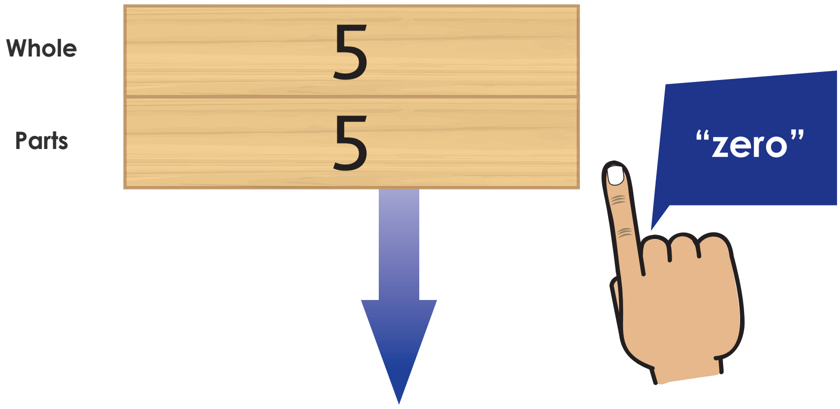
Both of these statements are mathematically INCORRECT. They do not build a robust understanding about addition and subtraction. More accurate responses to the error include

✓
- “Show me 3. Take away 7. What happened? You ended up less than zero, 4 less than zero.”
- “When taking away you need to put the whole first. What number is in the whole position? Remember, **Whole – Part = Part.**”

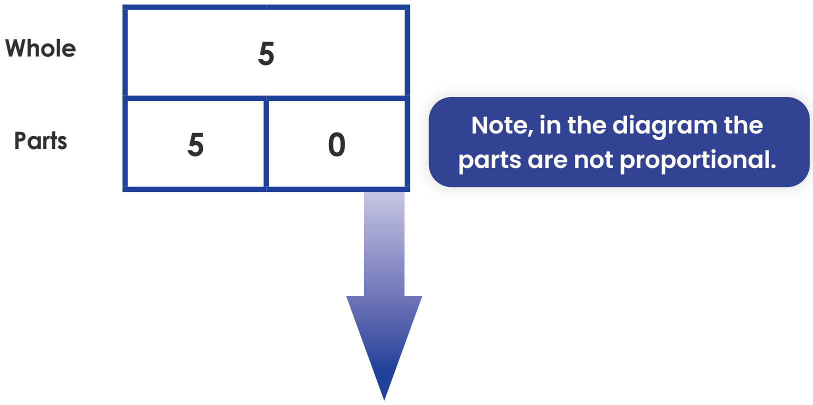
A Note On Zero

Bond Blocks are organised to represent part-part-whole. The **whole is always represented** when using Bond Blocks as per balancing two sets of manipulatives.

Representational Manipulative



Diagram



Abstract

5 + 0 = 5

0 + 5 = 5

5 - 5 = 0

5 - 0 = 5