Bond Blocks Support

Bond Blocks Support Book:

Teacher Notes

- The Evidence Base of Bond Blocks
- Building Equations: Using Part-Part-Whole
- Solving Missing Number Equations
- Solving Word Problems
- Solving Subtraction Using: Taking Away or Adding On
- Developing Calculation, Not Counting
- Using Part-Part-Whole: Desk Visuals
- Supporting Students with Learning Difficulties
- Instructions About Mathematical Language

Bond® Blocks
bondblocks.com

drpaulswan.com.au

Narelle Rice
& Dr Paul Swan

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Bond Blocks Support Book - Teacher Notes

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Authors: Narelle Rice and Dr Paul Swan

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The author may be contacted at: info@bondblocks.com.au
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Thank you for purchasing Bond Blocks.

We hope they help build

Curiosity,
Connections and
Confidence with maths.

- Narelle and Paul.

Contents

The Evidence Base of Bond Blocks

Defining 'Evidence-Based'

"Is Bond Blocks evidenced-based?" This is a great question and one we are often asked. However, it is also a term that is often misunderstood. We describe Bond Blocks as being "research-informed" and "using evidence-informed practice". We do this because strictly speaking, "evidence-based" is a research term that requires rigorous controlled trials. This is common in the medical field.

To answer the question "Is this evidence-based?" in an educational setting we refer to Professor Geoff Masters the CEO of the Australian Council for Educational Research (2018).

Evidence-based teaching involves the use of evidence to: (1) establish where students are in their learning; (2) decide on appropriate teaching strategies and interventions; and (3) monitor student progress and evaluate teaching effectiveness. (p. 4)

We recommend reading the entire article.
https://research.acer.edu.au/cgi/viewcontent.cgi?article=1335&context=research_conference

The salient points are that the medical profession, despite having access to controlled research studies also relies on other tests and evidence, including observations. Doctors try a course of treatment, monitor progress and alter the treatment according to observations. Masters explains that this comprehensive understanding of what an "evidenced based" approach is should also be applied to education.

"Policies and discussions of 'evidence-based teaching' sometimes overlook the importance of this broader, more integrated understanding of the role of evidence in teaching and learning." (p. 4)

Collecting Evidence

Bond Blocks has several tools built in to do what Masters (2018) defines as evidence-based teaching.
That is, collect data as evidence to:

- Establish where student are in their learning
- Monitor student progress
- Evaluate teaching effectiveness.

All Bond Block resources include assessment and monitoring resources to evaluate the effectiveness of teaching and learning.

The Bond Blocks Core Kit Test is used to identify specific areas of difficulty and is colour coded to relevant chapters of activities. It is re-administered annually to track progress over time. Results are entered into the Excel Results and Tracking Tool to monitor the progress of every student over time, identify school wide gaps in teaching and inform decision making.

All Bond Blocks resources contain monitoring tools at three levels:

- Tier One whole class teaching.
- Tier Two small group intervention.
- Tier Three individual student intervention.

Evidence-Informed Mathematics Teaching

Masters (2018) highlights that general, non-subject specific, evidence-based strategies "must be interpreted and implemented in the contexts of the subjects teachers teach... Teachers require evidence about the best ways to implement effective teaching strategies and interventions in subject-specific contexts" (p.5).

The following section outlines evidence informed principles that are embedded in both the "Bond Blocks Counting to 10 & 20" Kit and the "Bond Blocks Core Kit".

Mathematics Specific Evidence-Infromed Teaching Strategies

Bond Blocks incorporates the following key principles that have been identified by Sullivan (2001), and Anthony and Walshaw (2009), as effective for the teaching of **mathematics**.

Articulating Learning Goals*	The learning goals of every activity are provided.
Making Mathematical Connections	Connections are made between the concrete, representational and abstract elements of learning basic addition and subtraction facts through to pre-algebra and word questions.
Differentiated Teaching*	Activities are differentiated 'a little harder' and 'a little easier' alternatives.
Structuring Lessons*	The Bond Blocks session fits into standard lesson structures
Promote Fluency and Transfer	The goal of the system is to do away with the Bond Blocks in favour of automatic recall. Activities are structured to move from using the blocks, to diagrams, to finally using numbers only.
Mathematical Language	Mathematical language is specified in the teacher notes for every activity and is modelled in the teaching videos.
Assessment for Learning	Tools include a placement test that is used to ascertain prior knowledge and monitor progress and a variety of recording sheets to document observational notes.
Improving Teacher Knowledge	Clear succinct teacher notes are provided for every activity and concept along with Professional Learning opportunities.

***Key principles that are also identified as general High Impact Teaching Strategies (Victoria Department of Education, 2020).**

Bond Blocks incorporates these **general** High Impact Teaching Strategies:

Explicit Teaching	Explicit teaching is modelled in the videos that are provided for every activity.
Multiple Exposures	Bond Blocks activities are organised in cyclical chapters so that students return to the same concept, spaced over the teaching period.

Proficiency Strands

Sullivan's (2011) paper "Teaching Mathematics: Using research-informed strategies" is a review of research in the subject specific context of Mathematics. He draws on the work of Kilpatrick, Swafford and Findell (2001) who identify five intertwined strands of proficiency required for effective learning in mathematics.

It is from these five strands that four of the proficiency strands of the Australian Mathematics Curriculum (Australian Curriculum, Assessment and Reporting Authority 2022) https://v9.australiancurriculum.edu.au/ are based:

- Understanding
- Reasoning
- Fluency
- Problem Solving

Bond Blocks activities systematically incorporates each of these proficiencies into sequenced content as is illustrated in the **'Sequential Curriculum'** section.

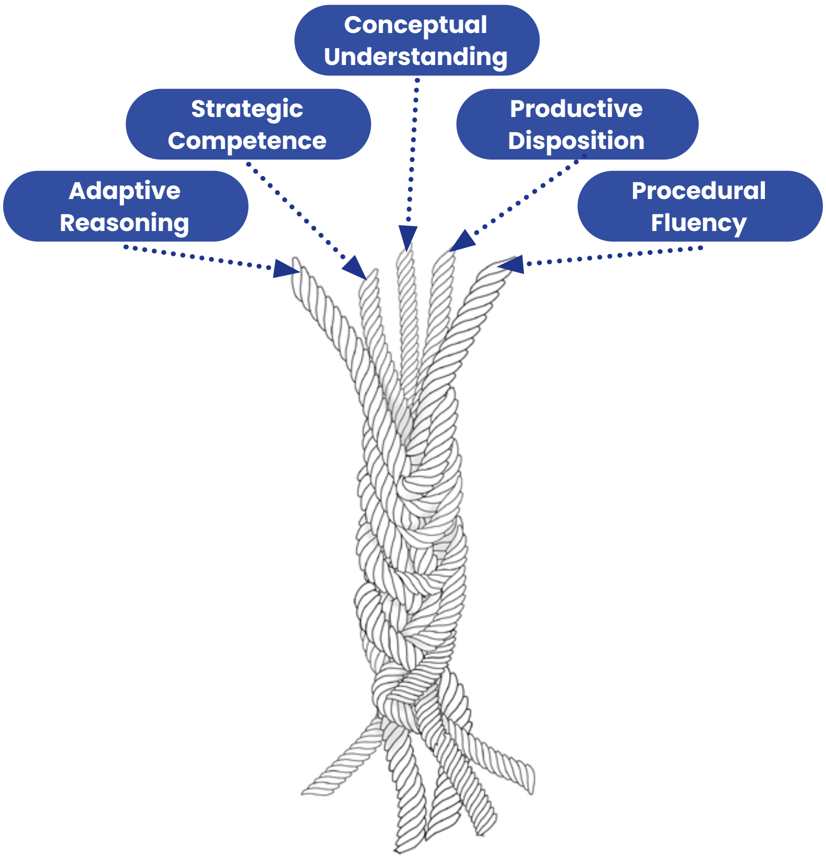


Figure 1. Five intertwined strands of mathematical proficiency.

Original diagram from "*Adding it up: Helping children learn mathematics*" by Kilpatrick, J., Swafford, J., & Findell, B.

The strand of **"productive disposition"** was not included in the Australian Curriculum however we believe it is essential for mathematics education. Sullivan (2011) quotes Watson and Sullivan (2008) in defining ***"productive disposition as a habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy"***. This is essential for both teachers and students. Sullivan (2011) stresses the importance of this for low-achieving students. The following features of Bond Blocks were built in to foster the development of productive disposition in teachers and students:

- Multiple activities highlight the innate beauty of mathematics as the **science of pattern**.
- Addition and subtraction are taught through understanding and making connections, so as students can **make sense** of them.
- The **mathematics** in every activity is clearly identified. Students know **why they are doing the activity**.
- The Bond Block test identifies what students do and don't know. Student then **set goals for their own learning**.
- The activities have been designed to have levels of engagement. Three-quarters of the activities are two player games. **Maths can be enjoyable.**
- Every topic has a set of Teacher Notes. These can be used by **teachers** to increase their **understanding of the mathematics**.
- Every activity is modelled in a video to support **teachers** in how to teach maths, increasing their **confidence teaching it**.

Sequentially Built, Cyclically Reviewed

Bond Block activities are sequenced based on prerequisite knowledge to ensure students have the required prior knowledge to build new understandings. Doing this reduces the risk students will rely on counting to calculate which is a major indicator that students will be at risk in mathematics. The activities are cyclically reviewed every chapter.

Mathematical Focus of each Core Kit Activity

Activity Chapter	Bonds	Fluency	Addition	Subtraction	Equation Building	Missing Number Equations	Word Problems	Calculating Strategies
2) Bonds of 5	6	7, 8, 9	10	11	12	13	14, 15	
3) Doubling and Halving to 10	16	17, 18						19, 20
4) Five Plus Bonds	21, 22	23	24	25				
5) Bonds of 10	26	27, 28	29	30	31	32	33	
6) Bonds of 6, 7, 8, 9	34	36, 37		35	38	39	40	
7) Ten Plus Bonds	41, 42, 43		44, 45, 47	48 (Set A)	46	48 (Set B), 49		50, 51, 52, 53, 54, 55, 56
8) Doubling and Halving to 20	57	58, 59, 60, 61						62, 63
9) Bonds of 11 to 20			64	65	66	67	68	69, 70, 71

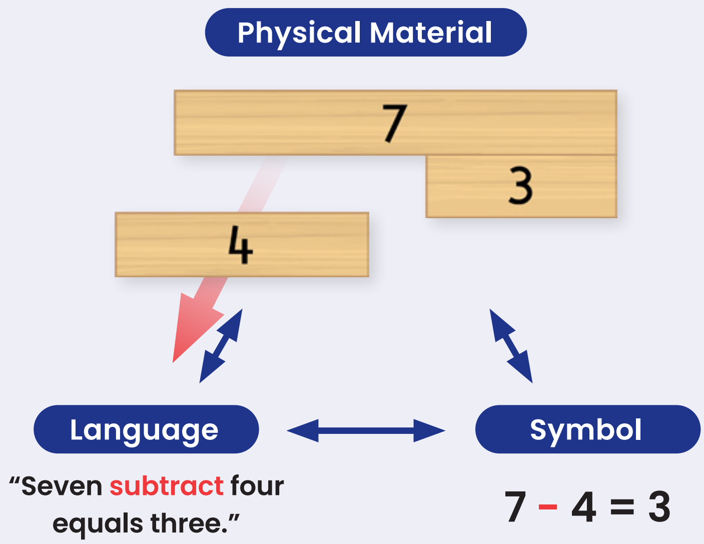
Numbers represent activity board numbers.

Mathematical Language and Connections

Mathematical understanding is strengthened when connections are made between the physical materials, mathematical language and symbols.

Core Kit

The way Bond Blocks are **positioned** (part-part-whole) and **moved** represents the operations of addition and subtraction. Students match the operation word, such as "subtract", with a corresponding action.



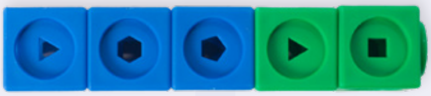
Concrete-Representational-Abstract

The Bond Block system is built using the **Concrete-Representational-Abstract approach**. This stems from the work of Bruner (1966). Bond Blocks are a **representational manipulative** that bridge the gap from concretely counting by ones to abstractly working with numbers and symbols.

Core Kit

Concrete

Most manipulatives can be counted by ones.



Many students don't progress from counting by ones.

Bridging the Gap

Representational

Bond Blocks are a physical manipulative to support bar-model maths.

The Missing Link to Abstract Calculation

Bond Blocks cannot be counted by ones.

- Learn number bonds (facts) in a self-correcting way.
- Represent and relate addition and subtraction using Part-Part-Whole.

Abstract

Adding and subtracting using numbers and symbols.

Part + Part = Whole

3 + 2 = 5

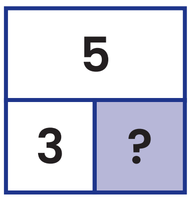
2 + 3 = 5

Whole - Part = Part

5 - 3 = 2

5 - 2 = 3

Extend with algebraic thinking.



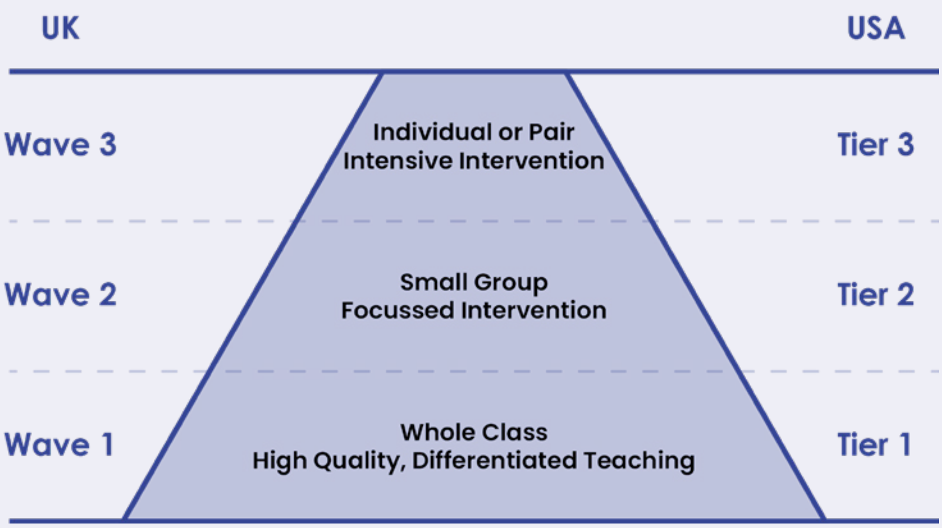
5 - ? = 3

3 + ? = 5

Evidence-Informed Intervention

Implementing Bond Blocks using Response to Intervention

The Bond Blocks System has been designed to be implemented at a whole school level. Implementation occurs at three different levels in line with a Response To Intervention process of instruction.



Bond Blocks is implemented at three different levels in line with a **Response To Intervention** process of instruction.

Firstly, the Bond Blocks Core Kit is implemented at a **tier one whole class** level as part of a whole school approach to teaching counting, addition and subtraction, including word problems and related algebraic thinking, from **Year 1 to Year 3**. Using Bond Blocks in these early years as a high-quality, differentiated teaching resource will reduce the numbers of students who require intervention.

Secondly, the Bond Blocks Core Kit is implemented at **tier two and three** as an **intervention program** for students from **Year 1 to 6** who have specific difficulties with foundational addition and subtraction. For example, students who count to add or subtract.

The GRATTAN Institute

For more information about implementing Bond Blocks refer to the implementation guides. The GRATTAN Institute's report (2023) ***"Tackling under-achievement: Why Australia should embed high-quality small-group tuition in schools"*** strongly endorses using tier two intervention in small groups of four students, as is recommended in the Bond Blocks implementation guide, as a cost effective way to help reduce the gap between advantaged and disadvantaged students. The report also recommends intervening as early as possible, before the gap increases. Bond Blocks targets the skills that are predictors of difficulty in maths from the first years of schooling, identified by research, and provides a high-quality differentiated resource to help close the gap.

Learning Difficulties Information Guide Numeracy

Using an **Response to Intervention** approach is endorsed by the State of Victoria Department of Education and Training (2019) *"Learning Difficulties Information Guide Numeracy"*. You can download this guide free from their site. One of our favourite quotes from this guide is, "***There is no 'one size fits all' approach to supporting students with learning difficulties in numeracy"*** (p. 14). Instead Intervention needs to be differentiated based on the needs of the learner. For this reason every Bond Block activity has differentiation options. This quote is also in line with Masters (2018) definition of evidenced-based teaching. The effectiveness of the intervention needs to be constantly evaluated for every student. This is why the Bond Block recording sheets for monitoring progress at a tier two and three level have significant space for recording anecdotes.

Response to Intervention Recommedations

Implementation of Bond Blocks for tier two and three intervention enacts the top four recommendations of the Institute of Education Sciences for RTI in Maths (Gersten et. al., 2009, p. 5).

Recommendation	Level of Evidence	Bond Blocks
Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.	Strong	• Explicit, video modelled teaching of every activity. • Systematically sequenced activities based on required prior knowledge and curriculum standards. • Verbalisation of mathematical process and content specified for every activity. • Mathematics is cyclically reviewed every chapter of activities. • Guided practice using gradual release model.
Interventions should include instruction on solving word problems that is based on common underlying structures.	Strong	• Word problem instruction uses underlying additive structures of part-part-whole and comparison problems, solved using Polya and the bar model. In depth teacher notes provided for professional learning.
Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas	Moderate	• Bond Blocks are a representational manipulative that is proportional. In each chapter students move from using the using physical Bond Blocks, to drawings of Bond Blocks, then to non-proportional part-part-whole diagrams.
Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.	Moderate	• Bond Blocks intervention specifies a minimum of four, ten minute sessions per week.

Research Informed

Each Bond Block resource has been designed to target the major predictors of maths difficulties, identified by research, with evidence-based teaching strategies. Therefore, Bond Blocks doesn't cover every area of the curriculum but focuses on:

- initial counting (Pre-Foundation and Foundation) using the "Bond Blocks Counting to 10 & 20" Kit and
- addition and subtraction (Year 1 to 3 level) using the "Bond Blocks Core" Kit.

> "Critical early quantitative competencies that children must possess to learn mathematics include an understanding of the relation between number words, Arabic numerals, and the underlying quantities they represent, as well as skill at fluently manipulating these representations; knowledge of the mathematical number line; and basic skills in arithmetic (i.e., skilled use of counting procedures, decomposition, and fact retrieval in problem solving). These skills are easily assessed in young children and many have been shown to be highly responsive to instructional interventions." (Geary, 2011, p. 15-16)

Core Kit

For students in Years 1 to 3 the major predictors of difficulties in maths are (Geary et al., 2009):

1. Persistent use of finger counting to add.
2. Lack of recall of basic facts (bonds).
3. Inefficient addition strategies such counting all.

The Bond Block Core Intervention Kit sequentially targets the teaching of all basic facts (bonds) used for addition and subtraction and the related strategies.

Intervention

The Core Kit can be used for Intervention with students who are not fluent with addition and subtraction of basic facts. It has been designed to support students with learning difficulties.

For more information please read these Bond Blocks Teacher Notes:

- "Developing Calculation, Not Counting". This outlines how to help students move on from counting when adding and subtracting to calculating without counting. This Teacher Note also contains information about characteristics of students with a specific learning difficulty in mathematics (Dyscalculia). bondblocks.com/tn-developing-calculation-not-counting
- "Supporting Students with Learning Difficulties". This Teacher Note outlines the range of features have been built into the Bond Block System to support students with learning difficulties.

Geary et al. (2009 p.4) states that when looking at students with a mathematical learning disability the "most consistent finding is that [they] show a deficit in the ability to use retrieval-based processes… and rarely use decomposition." He states that these students "rely on finger counting for more years" than students who do not have a disability in maths. Students who hold onto counting to solve addition and subtraction are at risk. These students do not move past the concrete stage of development for additive thinking.

The Core Kit systematically targets fluency with two-part bonds of wholes up to twenty and the development of partitioning based calculating strategies to help student move on from counting by one to add and subtract.

Prevention

It is worth noting that it is not only students with a mathematical learning disability who hold onto counting by ones to add and subtract. It is also mainstream students. Hopkins, Russo and Siegler in their 2022 paper "Is counting hindering learning?" found that over 30% of the Year 3 and 4 students they studied relied on counting to solve basic addition.

We found accurate min-counters [counting on from the largest number] represented over 30% of participants. These children were often incorrect when they were required to use retrieval for simple addition and were less flexible than most of the peers with mental computation strategies. (Hopkins et al., 2022, p. 52)

Their study involved 166 students "from three public primary schools in Melbourne, Australia…one school community was relatively advantaged… and two schools were similar to the Australian average" (p. 55). They surmised that "the percentage of children who do not consistently use retrieval to solve simple addition problems might well be even higher in lower-income areas" (p. 64).

This study involved simple addition. Whilst the number of students in Year 3 and 4 counting for addition is concerning, the number of students using counting for subtraction is even higher. In a different study Hopkins et al. (2023, p. 267) found "the prevalent use of decomposition strategies for single-digit problems that sum to between 10 and 20 and the use of counting strategies for corresponding subtraction problems".

Australian curriculums expect students to recall basic facts for addition and subtraction as well apply these to missing number problems in Years 2 and 3. The specific year varies from state to state depending on which version of the curriculum is being used.

The authors hypothesize that the following two factors are contributing to the significant number of tier one students not achieving the curriculum standard for addition and subtraction and are instead requiring intervention because they are dependent on counting:

- An overuse of discrete manipulatives at the Concrete stage of addition and subtraction that can be counted with one-to-one correspondence. Our observations are that when students are experiencing difficulty with addition and subtraction they are given more (albeit often different) manipulatives that can still be counted. This amplifies the problem. Our recommendation is the use of representational manipulatives, that cannot be counted by one, to bridge the gap from the concrete stage to the abstract stage of fluently adding and subtracting with numbers and symbols alone. This was the primary motivation for the creation of Bond Blocks.
- The importance of mathematical language at each of the Concrete-Representational-Abstract stages. Literacy has effectively upskilled teachers to use consistent whole school language when teaching English. Mathematics has failed to do this at a system wide level. For example, when asking what numbers that add to ten are called the following answers are often given in the same school: Bonds of 10, basic facts, compatible numbers, friendly numbers, friends of ten, rainbow facts. The lack of consistent school wide mathematical language contributes to ineffective teaching and learning. This is why every Bond Blocks activity states the mathematical language and models the correct use of it in explicit teaching on video.

Using the Core Kit are part of whole class, tier one teaching in Years One to Three will reduce the number of students who are stuck counting to solve addition and subtraction of basic facts. It also focuses on relationships between addition and subtraction to solve missing number and word problems. Prevention is better than cure.

We will conclude with our favourite quote from John Hattie (2016).

> **"Almost everything in published research works at least some of the time with some students. Our challenge as a profession is to become more precise in what we do and when we do it. Timing is everything, and the wrong practice at the wrong time undermines efforts." (p. 103)**

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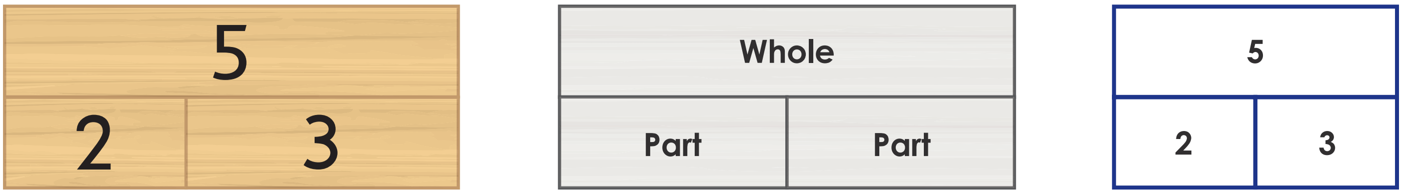
Dr Paul Swan (PhD) and Narelle Rice (BEd Hons)

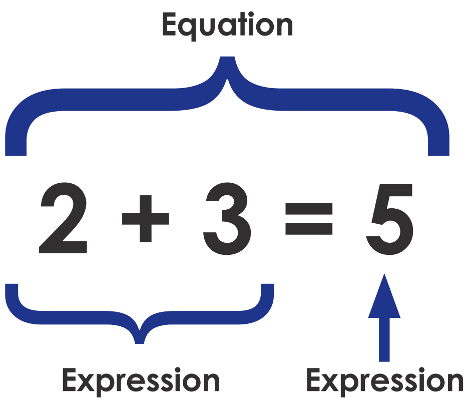
Building Equations: Using Part-Part-Whole

The concept of part-part-whole is used to teach addition and subtraction.

The part-part-whole diagram is a diagrammatic representation of the number bond. It is different to Bond Blocks in that it is not to scale.



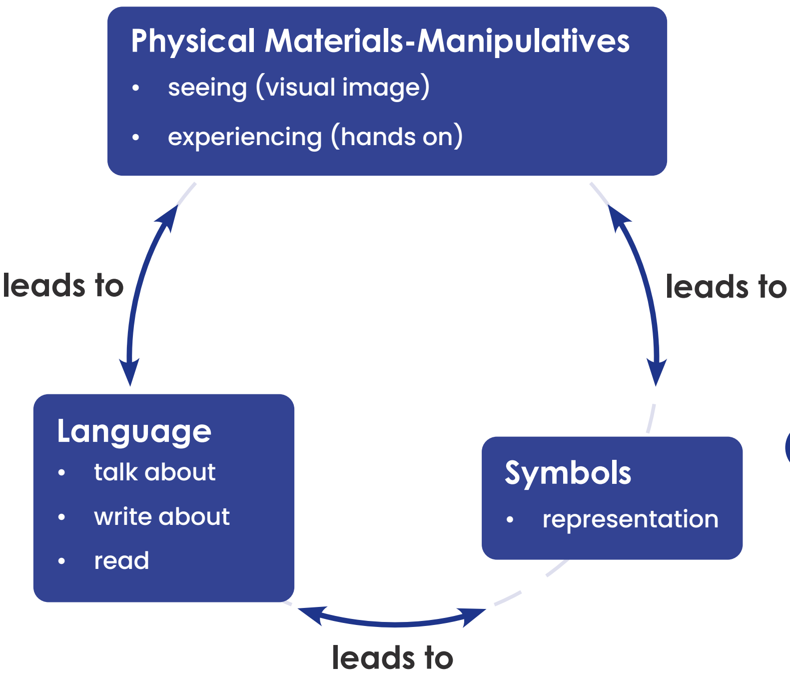
In order to develop a robust understanding of addition and subtraction the concept of "equals" need to be understood.

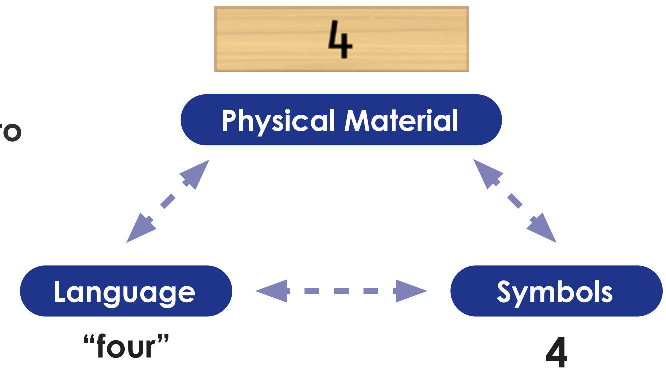


- **Equals** means "is".
- An **equation** contains an **equals symbol**. =
- Either side of an equation is an **expression**.

For example, in the equation 2 + 3 = 5 the expression on the left side of the equals symbol is 2 + 3 and the expression on the right side of the equals symbol is 5.

The following section outlines two different ways to represent equals using manipulatives. In each representation explicit links have been made between the manipulative, language and symbols.





Equals using One Set of Manipulatives

Frequently in primary schools “equals” is taught using one set of manipulatives.

- When adding, one amount of manipulatives is collected (one part), another amount is added (the other part), then the manipulatives are combined to make one set (the whole).

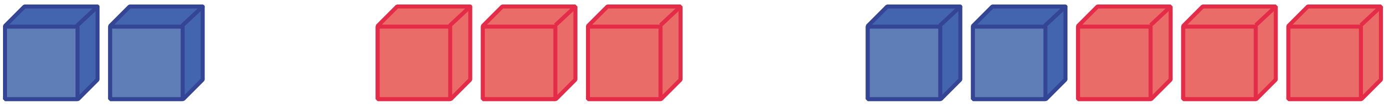
  Part + Part = Whole

- When subtracting, one set of manipulatives is collected (the whole), an amount of this set is separated (one part), leaving the remaining manipulatives (the other part).

  Whole – Part = Part

This example shows **2 + 3 = 5** which can be read as **“Two add three is five”**. It uses one set of 5 blocks.

2 + 3 = 5

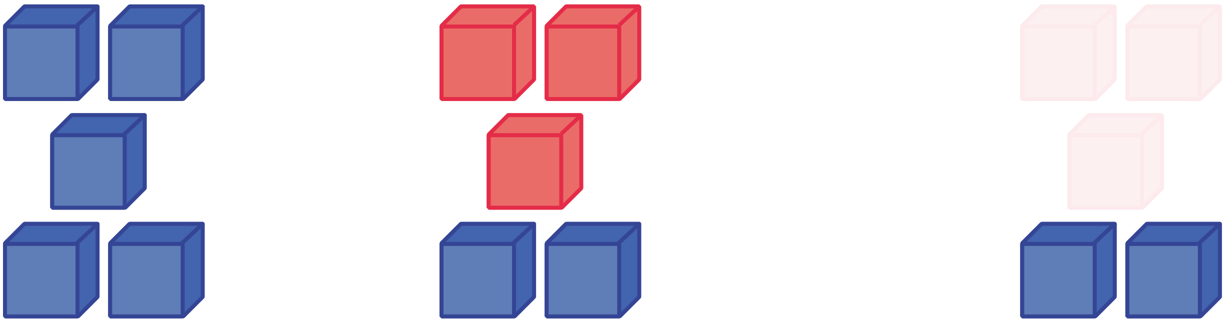


First collect two blocks. **Then add three more.** **There are five altogether.**

This example shows **5 - 3 = 2** which can be read as **“Five subtract three is two”**. It uses one set of 5 blocks.

If students are only exposed to using equals in this type of representation they can develop:

5 - 3 = 2



Start with 5 blocks. **Then subtract 3.** **This leaves 2.**

- Misconceptions. For example, thinking *5 = 2 + 3* is written ‘back to front’.
- Incomplete understandings. For example, only thinking about equals as an active “makes” which causes difficulties when solving static or comparison word problems.

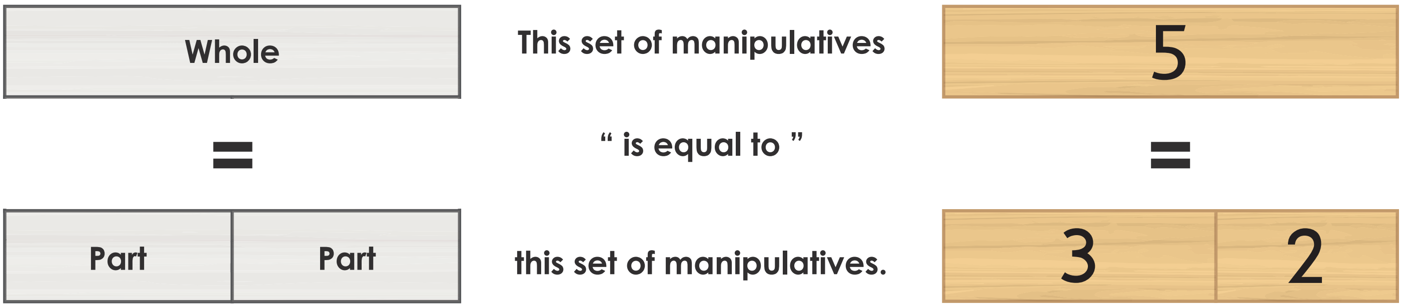
Equals Balancing Two Sets of Manipulatives

Equals can also be represented as balancing two sides of an equation. This involves using two sets of manipulatives, one set on each side of the equation, and adjusting the sets balance with each other. When this concept of equals has been used in Bond Blocks the phrase "is equal to" has been used.

The understanding of equals as balancing two sets is used to:

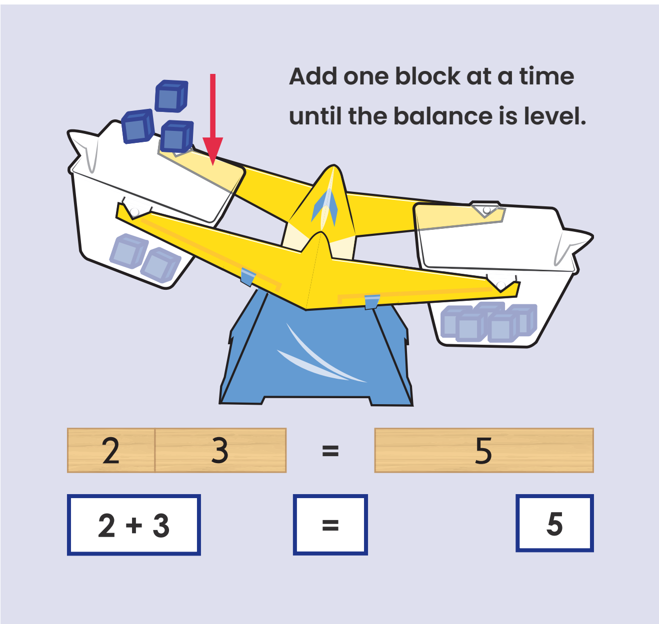
- Write equations in a non-standard way such as *2 = 5 - 3* or *3 + 2 = 2 + 3.*
- Solve comparison addition and subtraction word problems.
- Build a robust understanding of part-part-whole thinking.
- Lay the foundation for algebra.

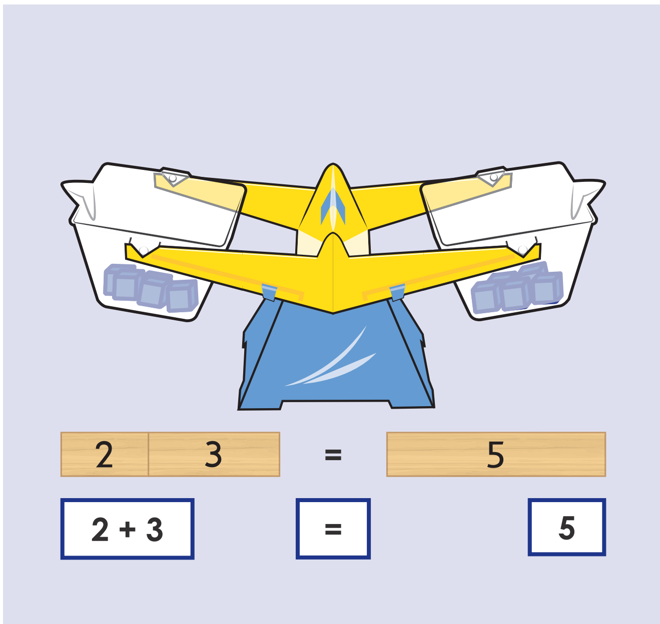
The Part-Part-Whole diagram, when used with to balance equations with manipulatives, uses two sets of manipulatives, one set on each side of the equation.



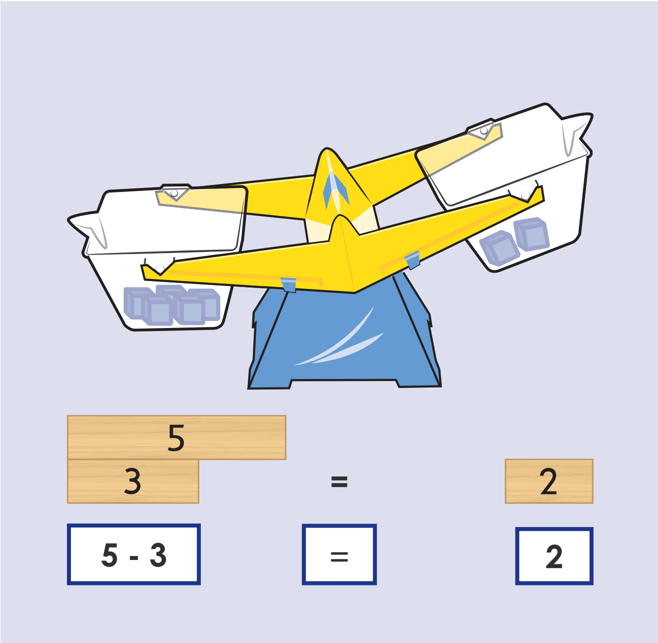
For activities involving balancing you will need to use plastic blocks because they are equally weighted. Bond Blocks are a linear system. **They will not balance because they are made of natural wood.**

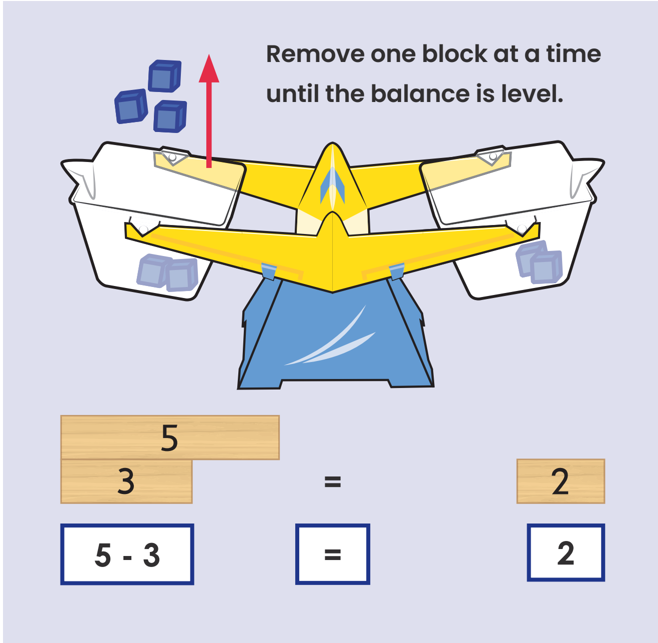
This example shows *2 + 3 = 5.* It uses two sets of 5 blocks.





This example shows *5 - 3 = 2.* It uses two sets of blocks.





Using the balance is an effective way to show the inverse between addition and subtraction, as indicated by the red arrows showing the adding or removing of blocks. The balance is a powerful visual when teaching inequality using greater than > or less than <.

Concrete Representational Abstract

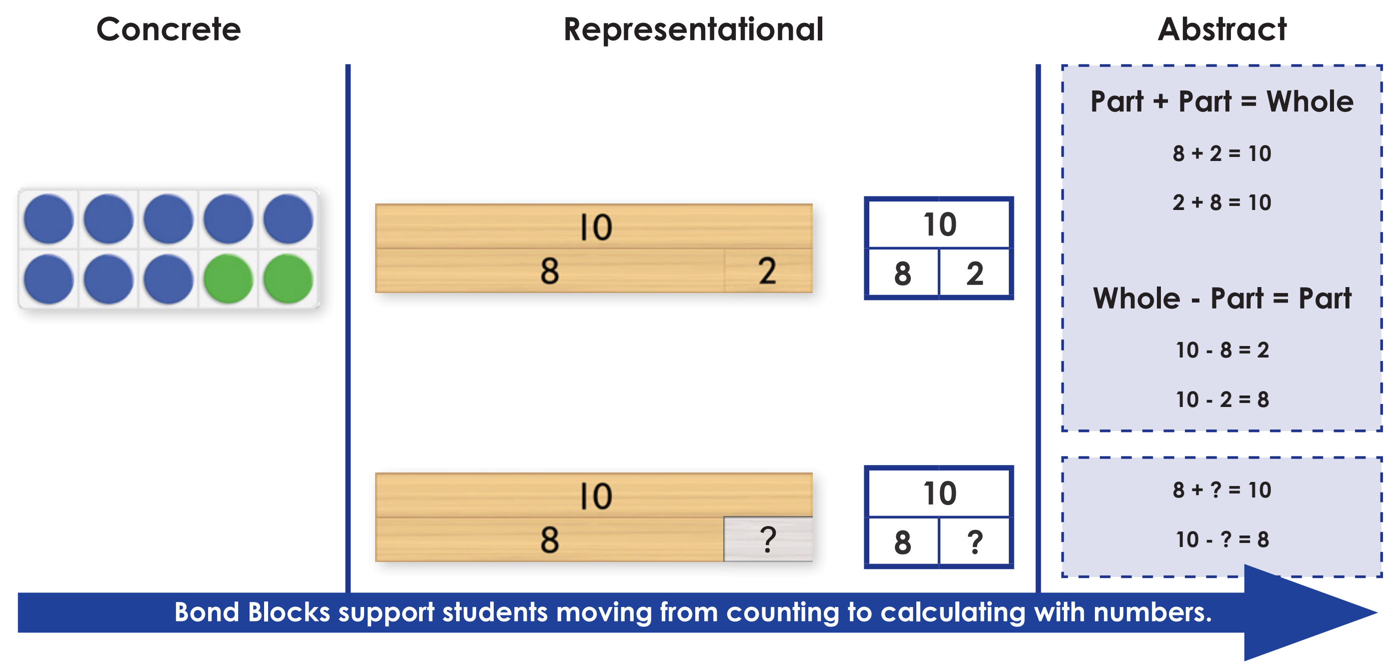
Bond Blocks fit into the teaching sequence after concrete manipulatives and before abstract written calculation. When teaching mathematics, students progress:

i) From a concrete understanding of equals as balance using manipulatives with one-to-once correspondence on a physical balance,

ii) To a representative understanding of balance using Bond Blocks,

iii) To an abstract understanding of balancing equations with numbers and symbols.

Students need to be exposed to multiple representations of equals to develop a robust understanding about equations.



Addition and Subtraction Using Part-Part-Whole

Students will need to be taught that for:

Addition

Part	+	Part	=	Whole

Subtraction

Whole	-	Part	=	Part

When adding the parts may be rearranged. Addition is commutative. Subtraction is not commutative.

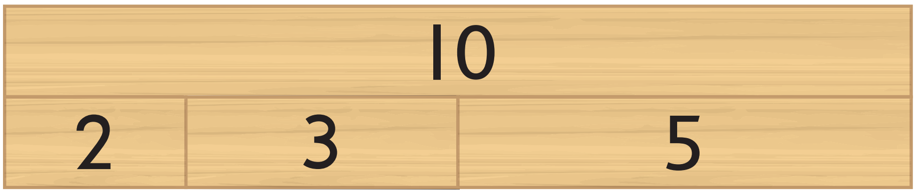
Whole	
Part	Part

7	
4	3

4 + 3 = 7   3 + 4 = 7   7 - 4 = 3   7 - 3 = 4

Extending Part-Part-Whole

Wholes can be made up of more than two parts. Here the whole of 10 has been partitioned into three parts.



Whole		
Part	Part	Part

10		
2	3	5

Activities may be made a little harder by using bonds of more than two parts.

Addition

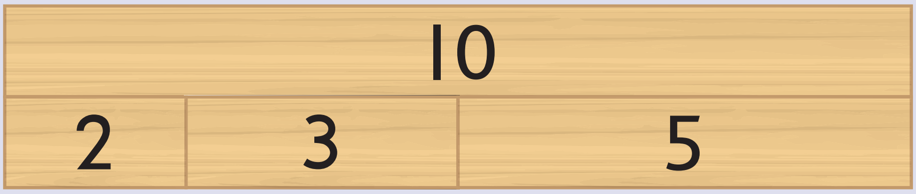
Part	+	Part	+	Part	=	Whole

Subtraction

Whole	-	Part	-	Part	=	Part

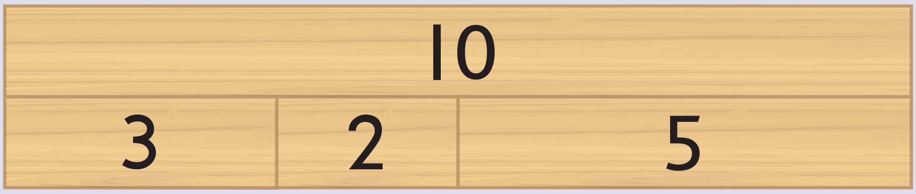
Parts can be rearranged because addition is commutative.

2 + 3 + 5 = 10



2 + 5 + 3 = 10

3 + 2 + 5 = 10



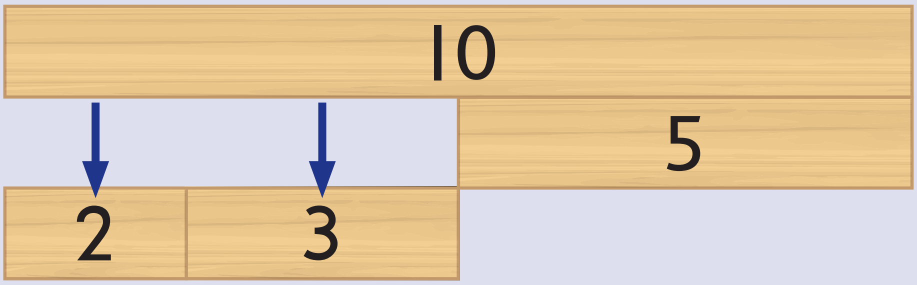
3 + 5 + 2 = 10

5 + 2 + 3 = 10

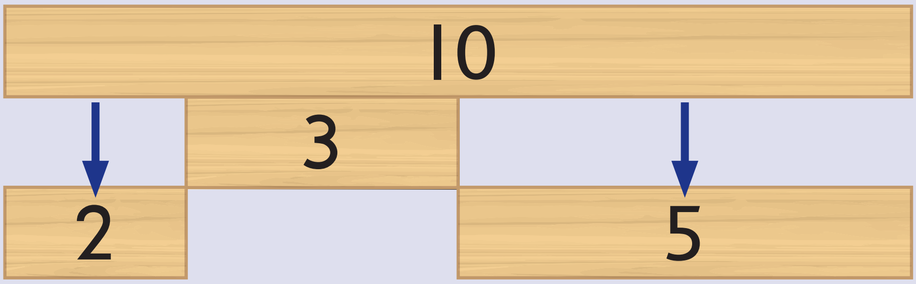
5 + 3 + 2 = 10

Parts have been subtracted from this part-part-whole representations.

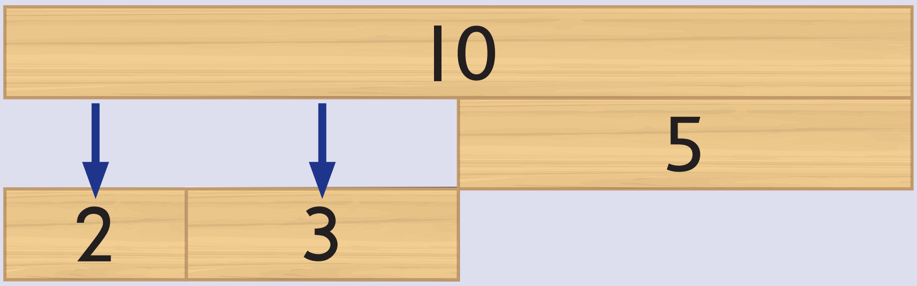
$10 - 2 - 3 = 5$



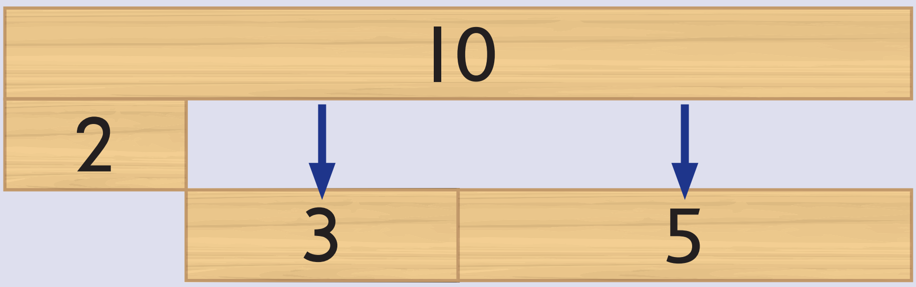
$10 - 2 - 5 = 3$



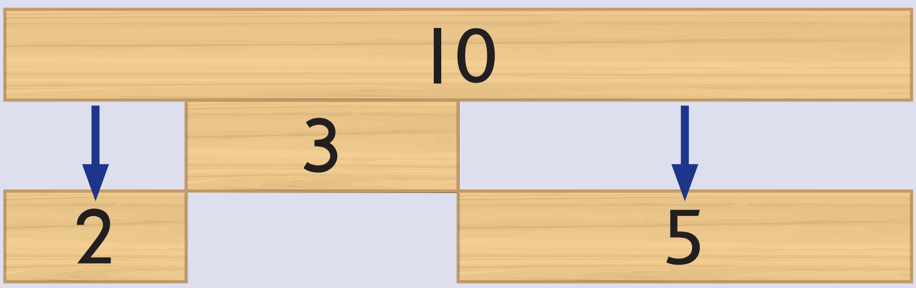
$10 - 3 - 2 = 5$



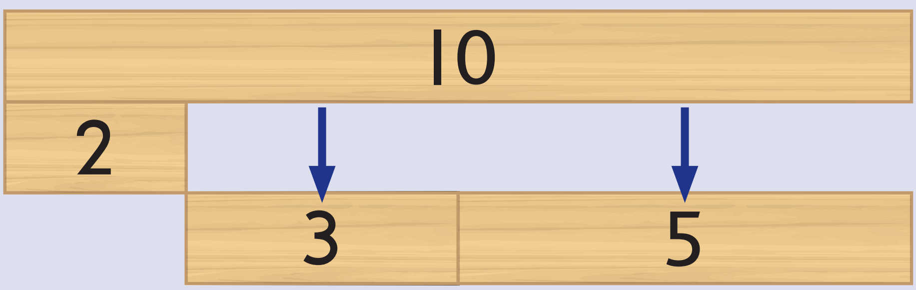
$10 - 3 - 5 = 2$



$10 - 5 - 2 = 3$



$10 - 5 - 3 = 2$



Atypical Arrangement using Part-Part-Whole

Addition and subtraction equations don't always have to be written as:

Part + Part = Whole

Whole - Part = Part

An alternative arrangement is:

Whole = Part + Part

Whole	=	Part	+	Part

Part = Whole - Part

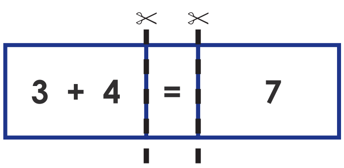
Part	=	Whole	-	Part

To teach this you will need:

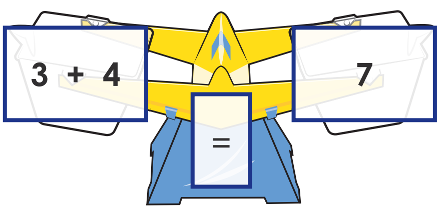
- one bucket balance.
- small ball of sticky tack to attach the card to the bucket balance.
- one piece of blank card approximately the same size as a bucket of the balance being used.
- a thick marker to write on the blank card.
- a pair of scissors.

Instructions

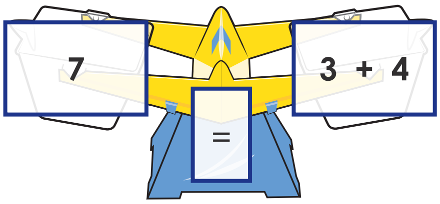
- Write an addition equation, in the typical order (part + part = whole), on the card.
- Cut the equation either side of the equal sign.



- Stick these to the buckets and fulcrum of the balance in typical order. Read from left to right.

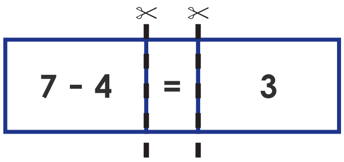


- Pick up the buckets and swap sides. This will produce the equation in atypical order. Reinforce that each side of the equals sign is read from left to right.

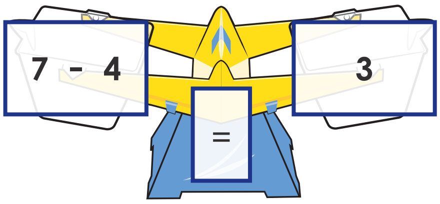


Repeat for subtraction

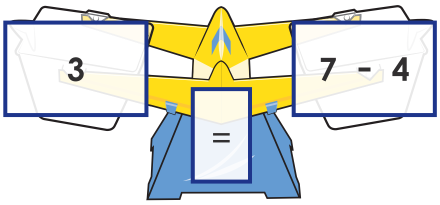
- Write an subtraction equation, in the typical order (whole – part = part), on the card.
- Cut the equation either side of the equal sign.



- Stick these to the buckets and fulcrum of the balance in typical order. Read from left to right.



- Pick up the buckets and swap sides. This will produce the equation in atypical order. Reinforce that each side of the equals sign is read from left to right.



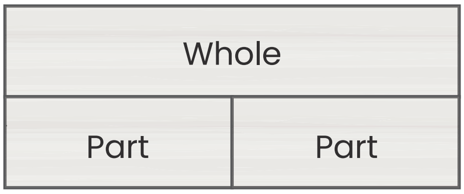
This method prevents students making the common error of writing the subtraction equation incorrectly when attempting atypical order.

Common Error.
Incorrect atypical arrangement.

7 - 4 = 3	$3 \neq 4 - 7$

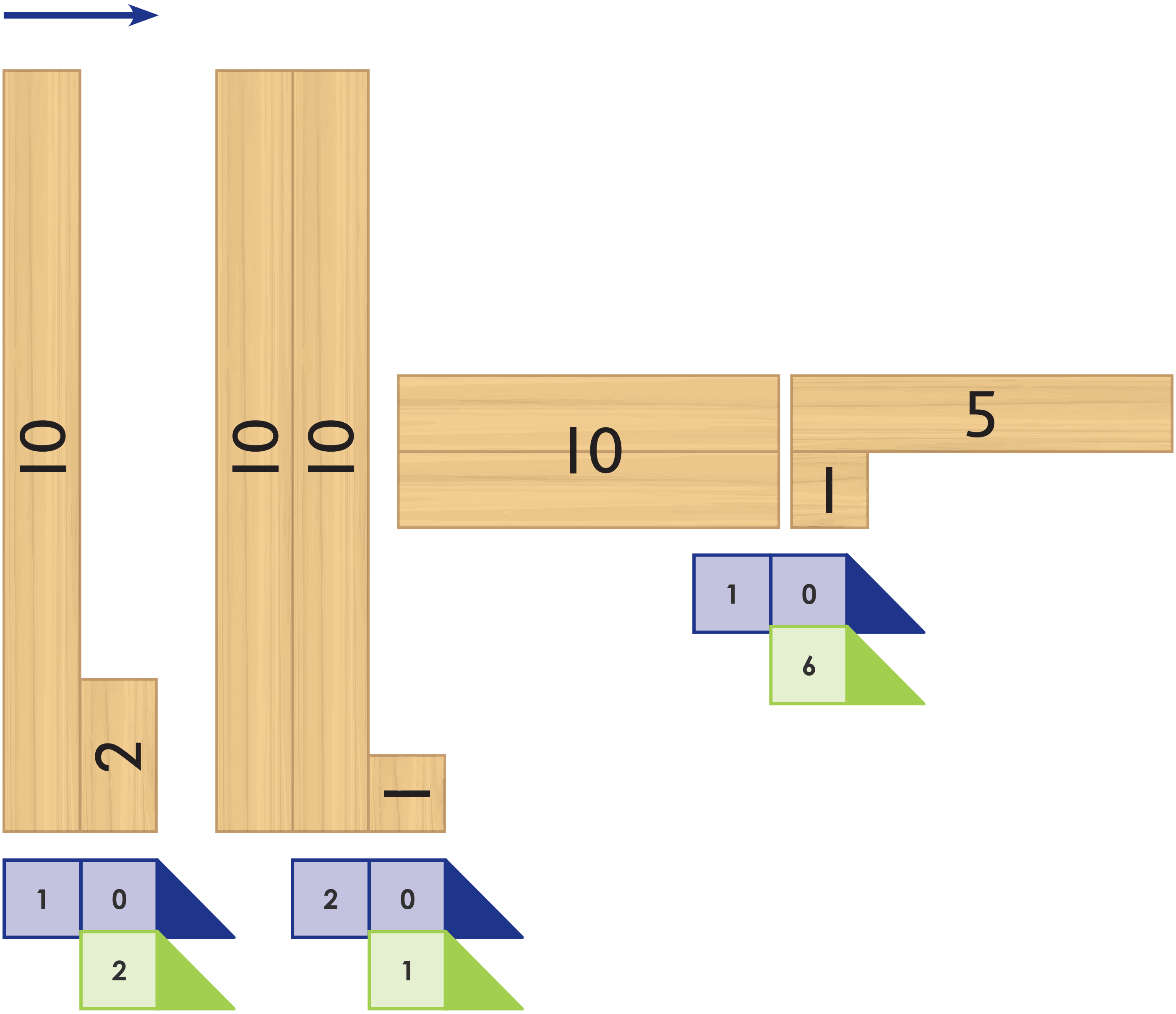
Block Placement

When using blocks to represent the whole and parts, place the whole on top and the parts underneath as per the part-part-whole diagram representation.



When placing parts generally the larger part is placed first, as long as this order is supported by the calculating strategy being used. Prior knowledge of the commutative property is needed to do this.

Work from left to right when placing blocks to support place value.

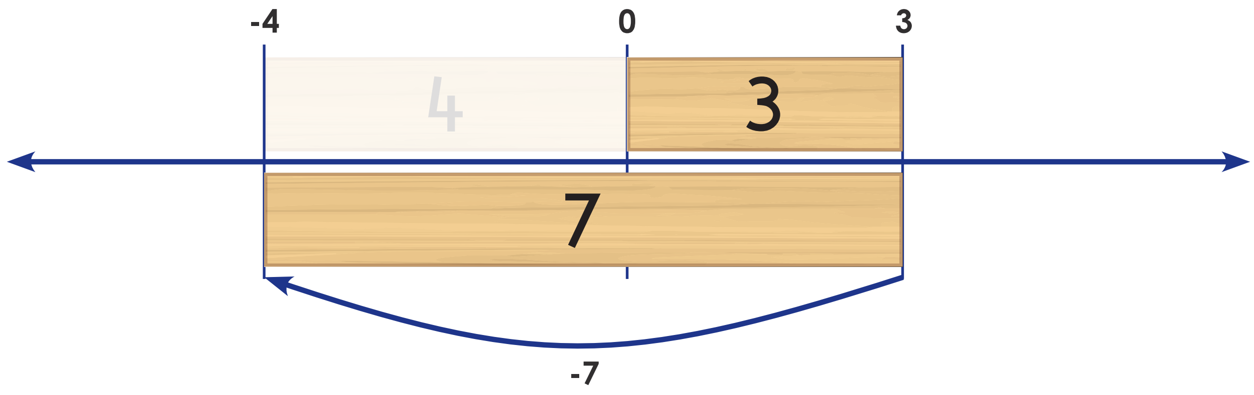


Once students demonstrate understanding and fluency they do NOT need to use the blocks.

A Note On Negatives

Often when students are creating subtraction equations they will incorrectly rearrange the whole and part. Students often write *3 - 7 = 4 which is incorrect*, when they mean *7 - 3 = 4* .

This can be demonstrated with Bond Blocks on an empty number line

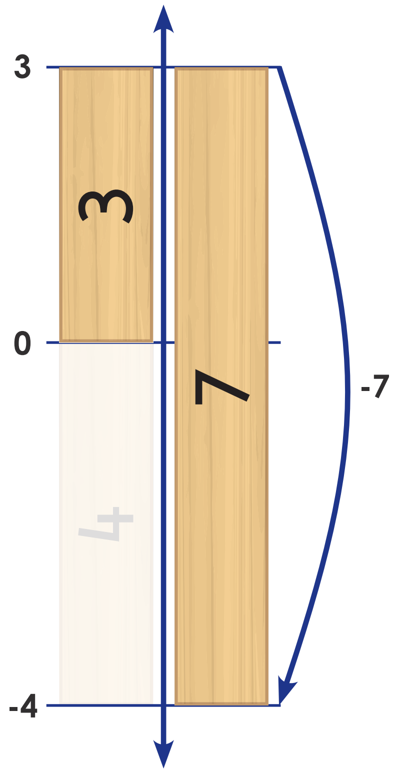


3 - 7 = -4 3 - 7 ≠ 4

For example, to demonstrate 3 - 7, start with 3 as the whole, on top of the empty number line. Subtract 7 by taking it away, placing it below the number line. The result is less than zero, four less than zero.

Some students find this easier to visualise if the number line is orientated vertically, as a thermometer.

eg. If it is 3 degrees and the temperature drops 7 degrees the temperature will be below zero.



When students write 3-7, but mean 7-3, teachers commonly try to correct this error by saying,

✗
- "Show me 3. You cannot take away 7" or
- "When taking away you need to put the bigger number first".

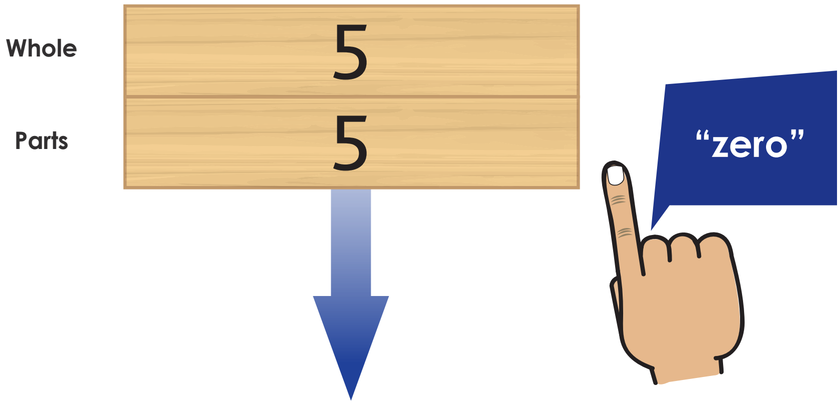
Both of these statements are mathematically INCORRECT. They do not build a robust understanding about addition and subtraction. More accurate responses to the error include

✓
- "Show me 3. Take away 7. What happened? You ended up less than zero, 4 less than zero."
- "When taking away you need to put the whole first. What number is in the whole position? Remember, **Whole - Part = Part.**"

A Note On Zero

Bond Blocks are organised to represent part-part-whole. The **whole is always represented** when using Bond Blocks as per balancing two sets of manipulatives.

Representational Manipulative



Diagram

Whole	5	
Parts	5	0

Note, in the diagram the parts are not proportional.

Abstract

5 + 0 = 5
0 + 5 = 5
5 - 5 = 0
5 - 0 = 5

Solving Missing Number Equations

Understanding Part-Part-Whole

Before solving missing number equations students need to:

- Understand addition and subtraction.
- Understand how to represent addition and subtraction using part-part-whole.
- Use these understandings to write addition and subtraction equations where the unknown is in the typical place to the right of the equals symbol.

Addition

Part Known	+	Part Known	=	Whole Unknown

$4 + 2 = \boxed{?}$

Subtraction

Whole Known	-	Part Known	=	Part Unknown

$6 - 4 = \boxed{?}$

Addition: Part Unknown

Whole Known	
Part Unknown	Part Known

6	
?	2

To solve missing number addition equations where the part is unknown students can either:

(i) Use addition.

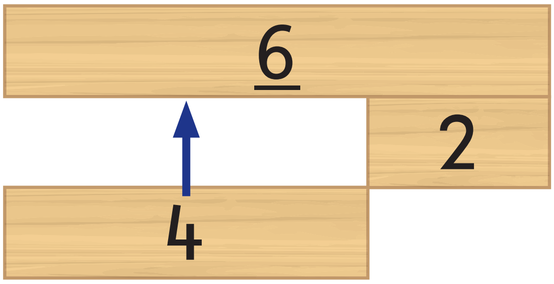
? + 2 = 6

Think, *"What joins with 2 to make 6?"*

Part Unknown + **Part Known** = **Whole Known**

Think, "*What joins with* this part to make the whole?"

Bond blocks can be used to support the calculation and check solutions.



(ii) Rearrange the equation, using part-part-whole to make a subtraction equation where the unknown is in the answer position.

? + 2 = 6 → 6 - 2 = **?**

Whole Known - **Part Known** = **Part Unknown**

The same process applies if the missing number is in the other part.

Whole Known	
Part Known	Part Unknown

6	
4	?

To solve missing number addition equations where the part is unknown students can either:

(i) Use addition.

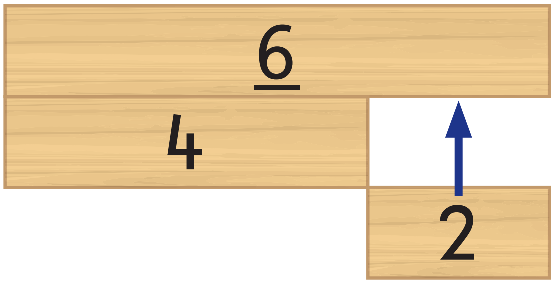
4 + ? = 6

Think, "*What joins with* 4 to make 6?"

Part Known + Part Unknown = Whole Known

Think, "*What joins with* this part to make the whole?"

Bond blocks can be used to support the calculation and check solutions.



(ii) Rearranging the equation, using part-part-whole to make a subtraction equation where the unknown is in the answer position.

4 + ? = 6 → 6 - 4 = ?

Whole Known - Part Known = Part Unknown

Subtraction: Part Unknown

Whole Known	
Part Unknown	Part Known

6	
?	4

To solve missing number subtraction equations where the part is unknown students can either:

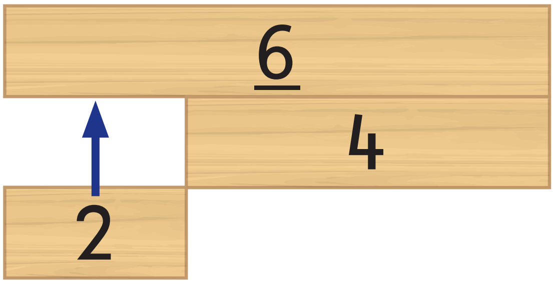
(i) Use addition.

6 - ? = 4

Think, *"What joins with 4 to make 6?"*

Part Known + Part Unknown = Whole Known

Think, "*What joins with* this part to make the whole?"



(ii) Rearrange the equation, using part-part-whole to make a subtraction equation where the unknown is in the answer position.

6 - ? = 4 → 6 - 4 = ?

Whole Known - Part Known = Part Unknown

Subtraction: Whole Unknown

Whole Unknown	
Part Known	Part Known

?	
2	4

To solve missing number subtraction equations where the whole is unknown students can either:

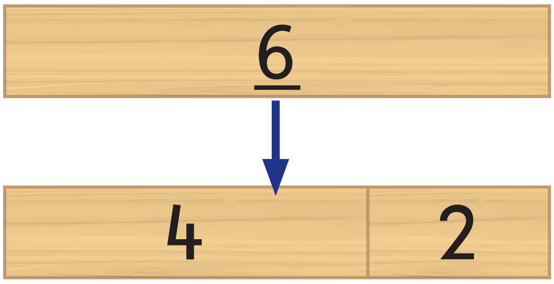
(i) Use addition.

? - 2 = 4

Think, *"Join the parts using addition to make the whole."*

Whole Unknown - Part Known = Part Known

Bond Blocks can be used to support the calculation and check solutions.



(ii) Rearrange the equation, using part-part-whole, to make an addition equation where the unknown is in the answer position.

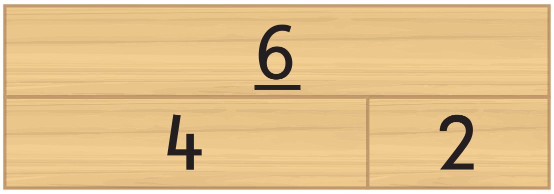
? - 2 = 4 → 4 + 2 = ?

Part Known + Part Known = Whole Unknown

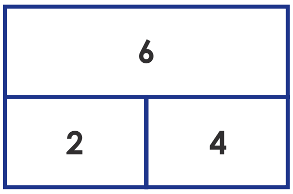
A Progression to Solve Missing Number Equations

Each of the strategies outlined to solve missing number equation have been based on this progression.

Concrete **Representational** **Abstract**



Bond Blocks manipulated to represent numbers, addition and subtraction.



Diagrammatic representation using part-part-whole.

$\boxed{?} - 4 = 2$

so $4 + 2 = \boxed{?}$

Calculation using numbers and symbols.

Bond Blocks support students to move toward calculating with numbers and symbols.

If students are experiencing difficulty backtrack one step. If they demonstrate understanding, help them to move forward one step using the concrete-representational-abstract progression.

Solving Word Problems

An Ineffective Approach

A common approach when teaching students to solve word problems is highlighting key words and assigning a mathematical operation to the word(s). For example, **"take away"** means subtraction needs to be used to solve the problem. Attributing operations to key words can be mathematically incorrect. In the following two examples a key word approach **does not work**.

"Josh had some toy trains. He took away five and gave them to his friend. He was left with three trains. How many trains did he have to start with?"

In this problem words such as **"took away"**, **"gave"** and **"left"** relate to addition.

The answer is found by

$$5 + 3 = \boxed{8}$$

"Mira has a collection of 23 toy figurines. This is 7 more than Kali. Write what you would enter into a calculator to find out how many toy figurines Kali has?"

In this problem the word **"more"** relates to subtraction.

The answer is

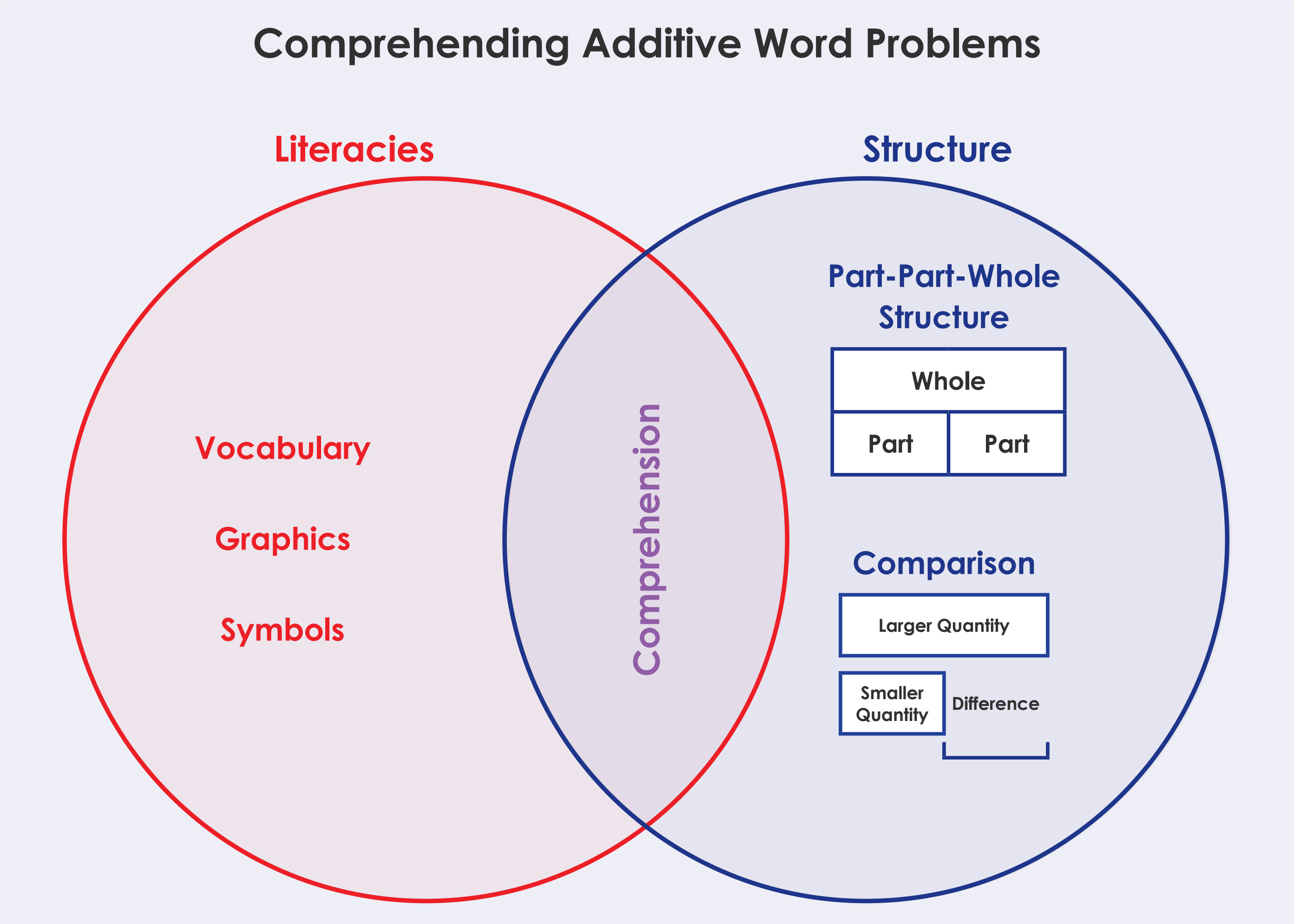
$$23 - 7 = \boxed{}$$

Powell and Fuchs (2018) state "Neither of these approaches – defining problems in terms of a single operation or linking key words to specific operations—has evidence to support its use". Instead students need to be taught to comprehend the situation described in the word problem.

"True comprehension occurs when one constructs a representation not only of the text itself, but also of the situation described by the text; both linguistic and mathematical knowledge." (Kitsch, 1986).

An Effective Approach

A more mathematically robust approach to solving word problems is to comprehend the words, numbers and graphics in the problem through the lens of problem structure/s.



Swan and Rice (2022)

Effective problem solving uses two scaffolds:

i. A general problem solving process such as the four steps devised by Polya (1954).

ii. A diagrammatic approach that represents the structure of the problem, such as a bar model.

i) Problem Solving Process

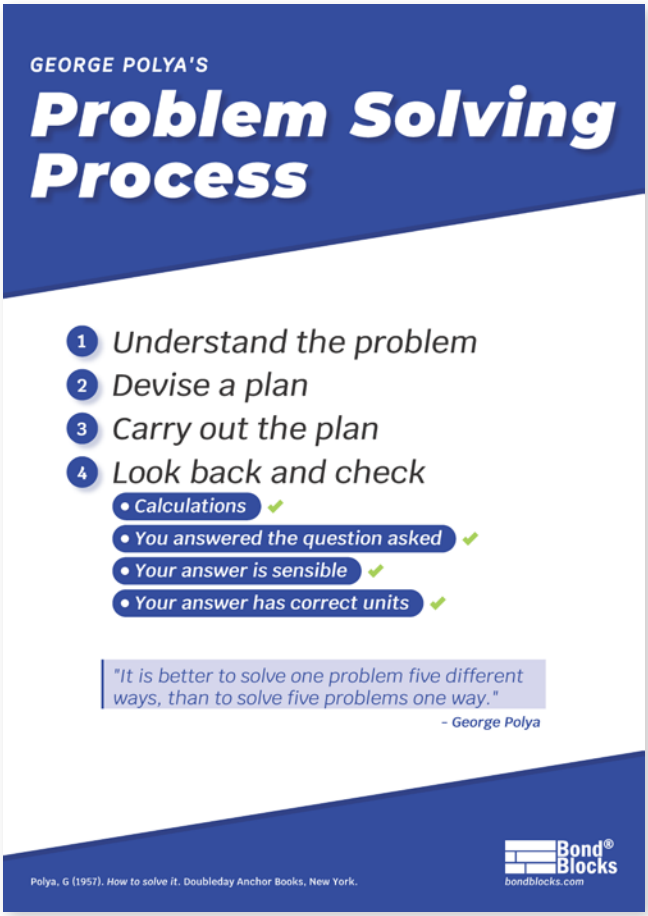
The four steps in Polya's problem solving process are:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

This problem solving approach is useful as a whole school approach for solving all different types of problems. Of which word problems are just one example. For very young children these steps may be shortened to **(1) Understand, (2) Plan, (3) Do, (4) Check**.

Polya's problem solving process is applied to Bond Blocks in more detail later in this document.

It is worth noting that **"Good problem solvers spend a bulk of time on Step 1 of Polya's method (understanding the problem and all relevant relationships) while novices rush to try a plan without really thinking through the plan's effectiveness."** Chapin, O'Connor & Anderson, 2009, p.97.



ii) Diagrammatic Approach

Using a diagram such as a bar model helps students do this. Bar model diagrams go by many names including strip diagrams (USA), tape diagrams (Japan), ribbon diagrams and Singapore Maths (introduced in the 1980s) [1]. Sawyer (1964) emphasised the importance of students drawing these diagrams to help them understand the mathematics. Bruner (1966) highlighted the importance of this in his stage of learning (Enactive, Iconic, Symbolic).

[1] *"The Ultimate Guide to Bar Modelling"*. Download from **www.thirdspacelearning.com**

Part-Part-Whole and Comparison Problems

Around the world there are multiple different ways of sorting additive (addition and subtraction) word problems into different structures (also referred to as "types", "schema based instruction" and "situations"). We have simplified these into two structures:

- Part-Part-Whole
- Comparison

All addition and subtraction word problems can be sorted into these two structures. It is important that students can identify each structure and the difference between them.

Typically students are exposed to part-part-whole problems more often than comparison problems, and find comparison problems more difficult than part-part-whole problems. For these reasons many students benefit from increased exposure to comparison problems.

Part-Part-Whole

Part-part-whole problems contain two parts that join together to make a whole.
Wholes can be partitioned into more than two parts.

Whole	
Part	Part

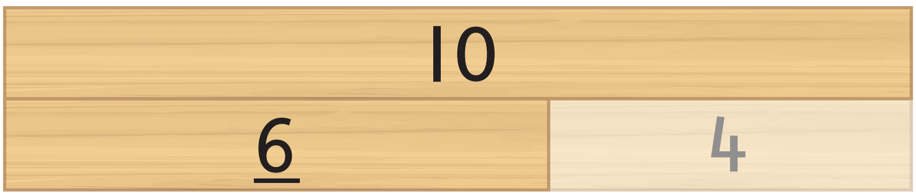
Part + Part = Whole

Whole - Part = Part

For example, Josh and his sister Grace combined their pocket money to buy a toy. Josh had $6. After Grace added her pocket money they had $10 altogether. *How much pocket money did Grace have?*

Whole Known	
Part Known	Part Unknown

Whole $10	
Josh's Part $6	Grace's Part $4



10 - 6 = 4

Grace had **$4** pocket money.

Comparison Problems

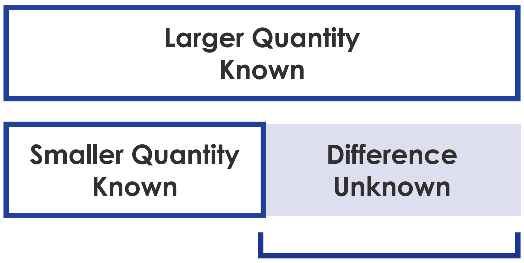
Comparison problems compare two quantities and use the difference between the two quantities. Comparison problems use comparative language such as "fewer/more", "less than/greater than".

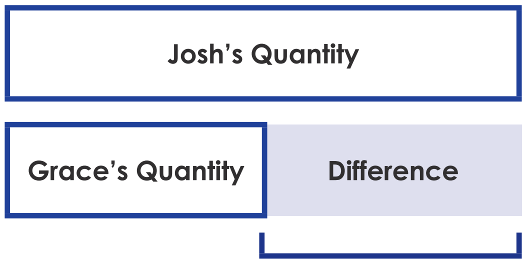
Smaller Quantity + Difference = Larger Quantity

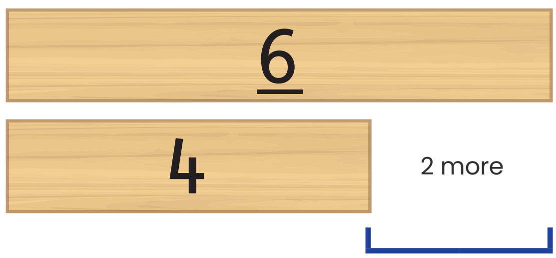
Larger Quantity - Smaller Quantity = Difference

Larger Quantity - Difference = Smaller Quantity

For example, Josh had $6. Grace had $4. *How much more money did Josh have compared to Grace?*







$$6 - 4 = \boxed{2}$$

Josh had **$2** more than Grace.

Active and Static Word Problems

Part-Part-Whole and Comparison word problems can be written in either an active or static way.

Active problems have an action where the parts (in a part-part-whole structure) or quantities (in a comparison structure) interact with each other. For example, in the following two questions the girls join the boys on the mat. This makes them active questions.

(Active / Part-Part-Whole)

There were 4 boys sitting on the mat. Six girls came and sat down to join them. How many children were sitting on the mat altogether?

Part-Part-Whole word questions written in an active way are sometimes referred to as Change or Join questions.

(Active / Comparison)

There were 4 boys sitting on the mat. Six girls came and sat down to join them. How many more girls were there compared to boys sitting on the mat?

In **static problems** the parts (in a part-part-whole structure) or quantities (in a comparison structure) do not interact with each other. For example, in the following two questions the girls and boys are sitting on the mat. This makes them static questions.

(Static / Part-Part-Whole)

There were 4 boys and 6 girls sitting on the mat. How many children were sitting on the mat altogether?

(Static / Comparison)

There were 4 boys and 6 girls sitting on the mat. How many more girls were there compared to boys sitting on the mat?

Classifying questions as active or static is unnecessary for students because this information is not needed to solve the problem. Whereas identifying the question as a part-part-whole or comparison structure is.

However, it is helpful for teachers to be aware of active and static ways of presenting word problems. Typically students are exposed to active problems more often than static problems. Active problems also tend to be easier for students to solve because they can be acted out at different levels (concretely, representationally or abstractly in your minds' eye). This makes comprehending the problem and identifying the needed operation easier. For these reasons many students benefit from increased exposure to static problems.

Missing Number Positions

The missing number, or 'unknown', can be placed in any position within the equation related to the word problem. This one example, "A fish tank contained 10 fish. Six were gold, four were black," can be used to write word problems where the missing number is placed in a **variety** of positions.

10	
6	4

Typically students are exposed to word problems where the missing number or 'unknown' is isolated on one side of the equation. For example, addition as *6 + 4 = ?* and subtraction as *10 - 6 = ?* These types of questions are shaded darker in the six examples below.

Finding the unknown in a position other than the answer position is often more difficult for students. These questions are shaded lighter in the six examples below. Students benefit from increased exposure to word problems where the missing number is in different positions.

Unknown	
6	4

10	
Unknown	4

10	
6	Unknown

Static Examples

$6 + 4 = \boxed{?}$

There were 6 gold fish and 4 black fish in a tank. The tank had no other fish.

How many fish were in the tank?

$\boxed{?} + 4 = 10$

There were some gold fish in a tank and 4 black fish. Altogether there were 10 fish.

How many were gold?

$6 + \boxed{?} = 10$

There were 6 gold fish in a tank and some black fish. Altogether there were 10 fish.

How many were black?

Active Examples

$\boxed{?} - 6 = 4$

There were some fish in a tank. Six gold ones were moved to a pond. This left 4 black fish in the tank.

How many fish were in the tank to start with?

$10 - \boxed{?} = 4$

There were 10 fish in a tank. Some were gold. The gold fish were moved to a pond. This left 4 black fish in the tank.

How many fish were gold?

$10 - 6 = \boxed{?}$

There were 10 fish in a tank. Six gold ones were moved to a pond. The fish left in the tank were all black.

How many were black?

Supporting Students Who Have Difficulty

Identify the stage of difficulty

Newman (1983) identified five stages students progress through when solving word problems. Different errors arise at each stage and are identified by interview cues.

Newman Error Analysis Stage	Newman's Interview Cue	Related Stage in Polya's Problem Solving Process
Reading and Decoding The student reads the problem and decodes words and symbols.	Please read the question to me. If you don't know a word, leave it out.	**Understand the problem**
Comprehending The student make sense of what they have read.	Tell me what the question is asking you to do.	
Transforming The student 'mathematises' the problem; that is, works out what maths is needed to be done.	Tell me how you are going to find the answer.	**Devise a plan**
Processing The student does the maths.	Show me what to do to get the answer. 'Talk aloud' as you do it, so that I can understand how you are thinking.	**Carry out the plan**
Encoding The student records their final result appropriately.	Now, write down your answer to the question.	**Look back and check**

Numberless problems

Exposure to "numberless problems" (Bushart, 2014), prior to solving word problems with numbers, can significantly increase understanding.

There are two distinct steps we need to take when solving a word problem. First we must work out the mathematical relationship between the quantities and, second, we must operate on the quantities. The first step is totally independent of the actual size of the quantities. (Askew, 2019, p. 56)

By engaging with word problems that are numberless students focus on understanding the relationships between the numbers. Numbers are introduced in gradual steps so as the students "talk about the relations between quantities without knowing the actual quantities. As specific numbers are gradually introduced so the reasoning about what to do with those numbers emerges" (Askew, 2019, p 56).

For example, this word problem could be scaffolded, beginning with a numberless word problem, using the following steps:

? + 4 = 10

There were some gold fish in a tank and 4 black fish. Altogether there were 10 fish.
How many were gold?

Steps:

1. There were fish in a tank. Some were gold. The other fish were black.
2. There were 10 fish in a tank. Some were gold. The other fish were black.
   (This is an ideal place to pause and investigate possibilities.)
3. There were 10 fish in a tank. Some were gold. The other 4 were black.
   How many fish were gold?

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Solving Subtraction: Using Taking Away or Adding On

Subtraction using Number Lines

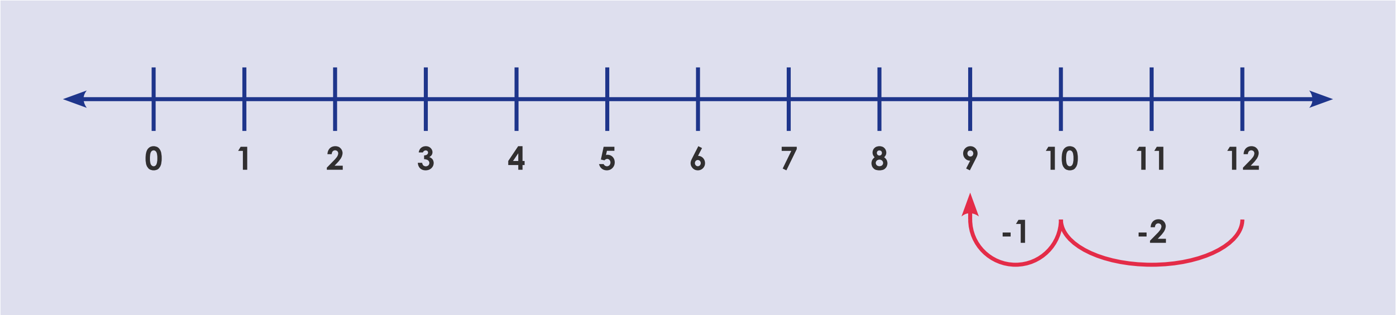
Subtraction can be solved be solved using a number line by either:

- Taking away, moving left (counting backwards) or
- Adding on, moving right (counting forwards).

For example,

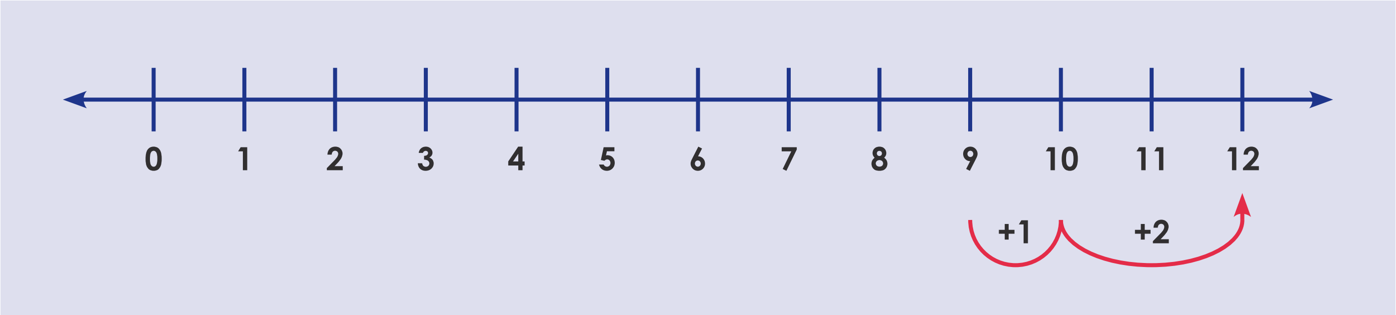
Taking Away

12 − 3 = ? can be solved by starting with the whole of 12, then moving left to take away the known part of 3.



Adding On

12 − 9 = ? can be solved by starting with the known part of 9, then moving right, adding on the unknown part, to make the whole of 12.



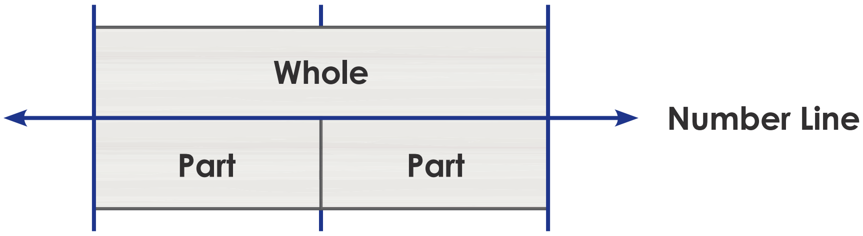
Strategy Development

Typically children's brains find adding on easier than taking away. Adding on relates to counting forwards. Taking away to counting backwards. For this reason subtraction as take away is more suited to numbers where the part being taken away is relatively small, reducing the amount of backwards thinking. For example, taking away is efficient for *12 - 3* whereas adding on is efficient for *12 - 9.*

Counting on or back is an initial strategy used to subtract. However, students should **not** count on or back more than **1, 2 or 3** (Booker et al., 2014). This includes jumps of one on a number line. Alternative strategies need to be taught.

Subtraction using Bond Blocks on Empty Number Lines

Bond Blocks are placed on empty number lines using the part-part-whole diagram arrangement.



Use Bond Blocks to solve subtraction on empty number lines by placing the:

i. **Whole** first, above the number line.
ii. **Known Part** next, below the number line.
iii. **Unknown Part** (which is the answer) last, below the line.
This part will fill the gap below the line to make the parts equal in length to the whole.

Subtraction can be solved be solved using an empty number line by either:

- Taking away, moving left or
- Adding on, moving right.

Once students are fluent solving subtraction using taking away and adding on encourage them to:

- Pause before beginning to solve a subtraction question.
- Consider the size of the whole and known part in relation to each other.
- Choose whether taking away or adding on is more efficient.

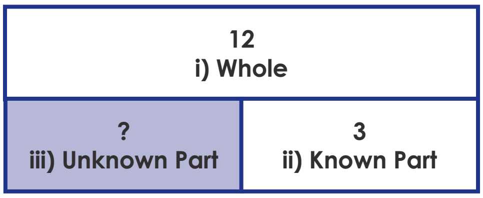
Taking Away

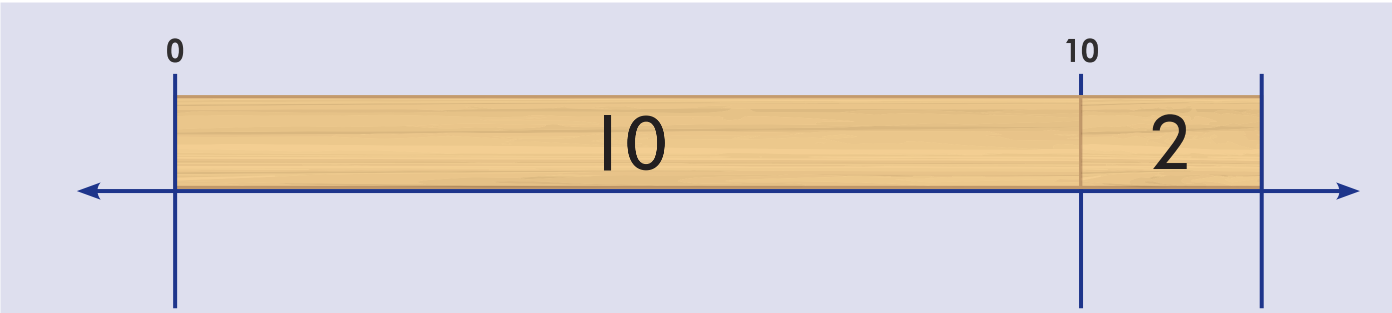
When using Bond Blocks to solve subtraction by taking away place the blocks in this order:

i. Represent the **whole.**
ii. Represent the **known part** being taken away. Start at the whole, place blocks moving left.
iii. Represent the **unknown part** that is the answer. Fill the gap below the line to make the parts equal in length to the whole.

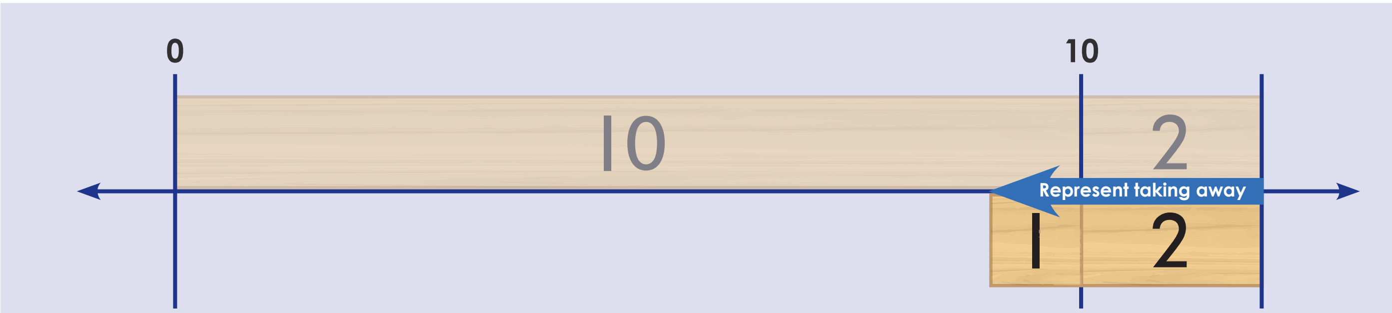
For example,

12 - 3 = ?

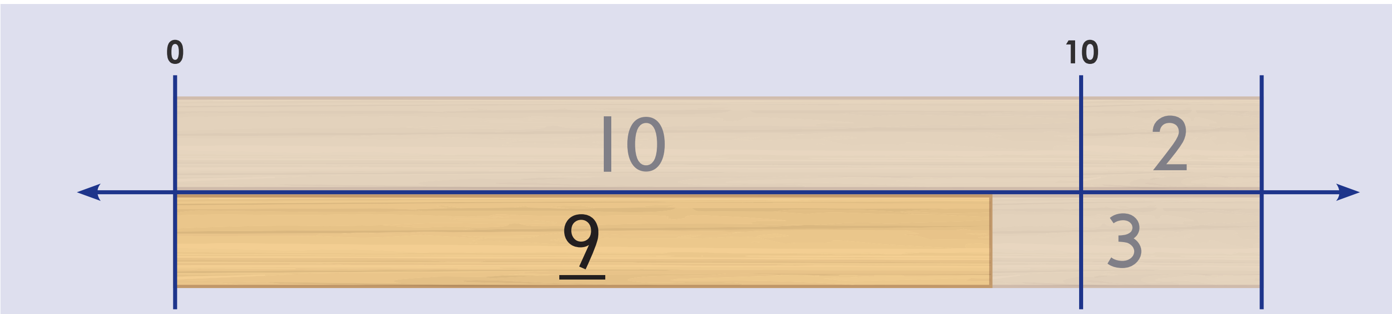




i) The whole of 12



ii) Take away the known part of 3



iii) Equals the unknown part of 9.

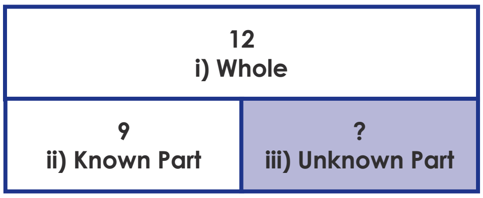
Adding On

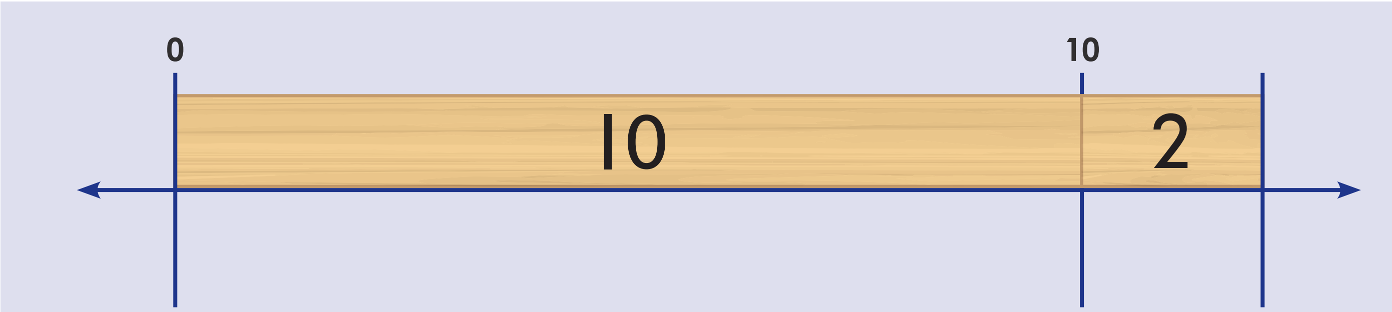
When using Bond Blocks to solve subtraction using adding on place the blocks in this order:

i. Represent the **whole**.
ii. Represent the **known part**.
iii. Start at the **known part**, add on blocks moving right, to make the parts equal to the **whole**. The amount added on will be the **unknown part** (answer).

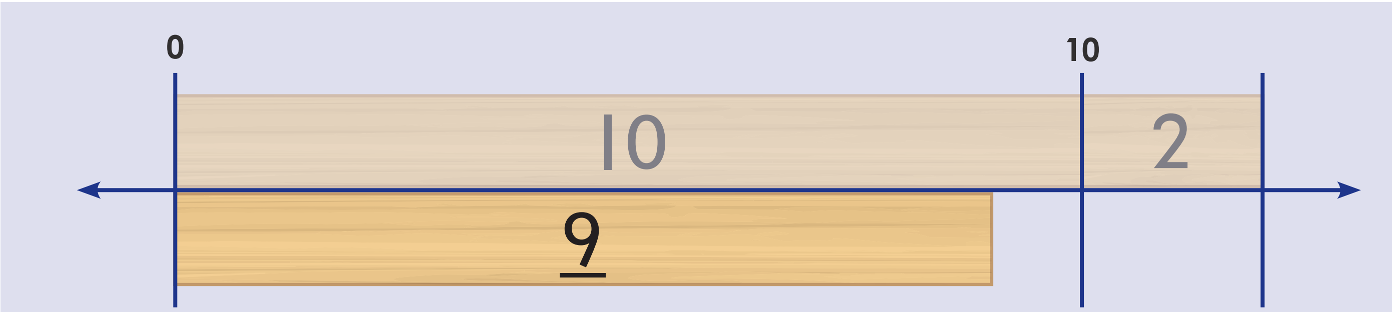
For example,

12 - 9 = ?

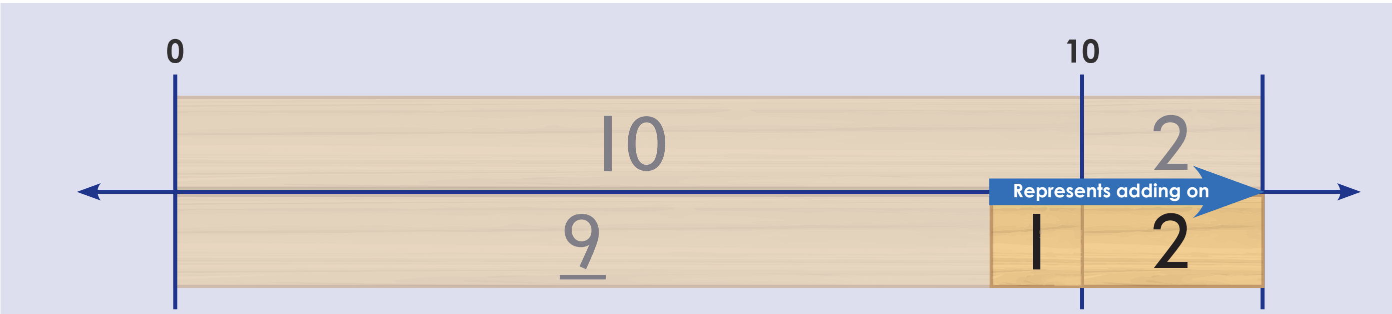




i) The whole of 12



ii) The known part of 9



iii) Add the unknown part of 3

References

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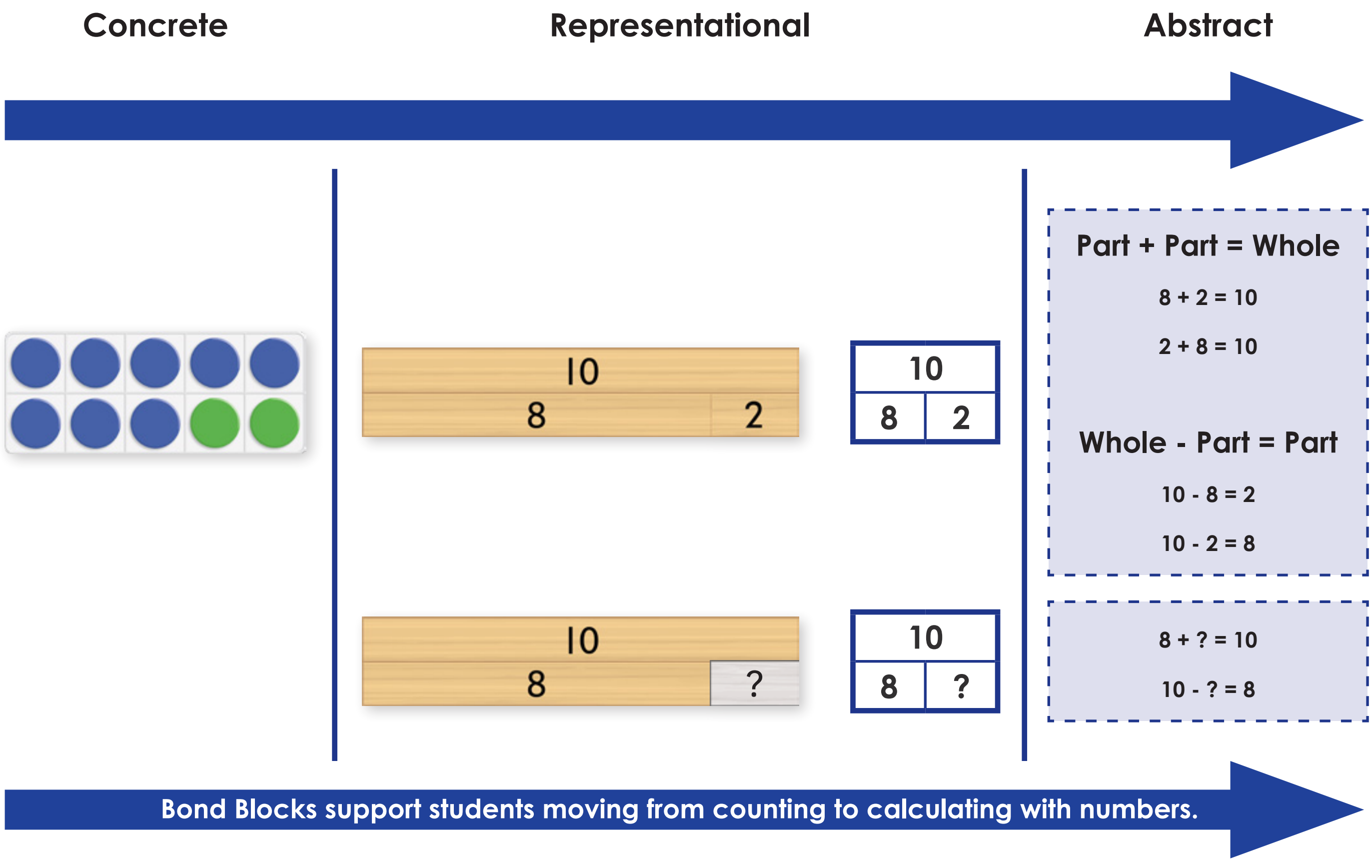
Developing Calculation, Not Counting

Developing Counting Principles

Initially Bond Blocks can be used in conjunction with discrete countable objects such as counters whilst students develop the principles of counting (Gelman & Gallistel, 1978).

1. The stable order principle:
   - Number names are said in the conventional order.
2. The one-one principle:
   - Each item is counted once as the corresponding word is said.
3. The cardinal principle:
   - The last number said indicates the total for the group.
4. The order-irrelevance principle:
   - Counting left to right, right to left or in a scattered arrangement does not change the quantity.
5. The abstraction principle:
   - Physical items that differ in type, colour and size can be counted.
   - Things that cannot be touched such as sounds and ideas can be counted.

These principles need to be grasped before the counters are removed and only Bond Blocks are used. This follows Bruner's (1966) approach of moving from:



Counting or Calculating

The Bond Blocks system makes the distinction between counting and calculating clear. The Bond Blocks Core Kit is used to help students move from counting to adding and subtracting (without counting).

When first learning to add and initial strategy students first learn is counting on from the larger number (zero, one, two or three). Similarly, when beginning to subtract, students initially use the strategy of counting back (zero, one, two or three). This strategy needs to be replaced by more efficient strategies. For this reason, it is recommended that when teaching students should not count on or back more than three.

If students do count on or back more than three to solve addition or subtraction the teacher can:

i. Provide different manipulatives.
ii. Model a different strategy.
iii. Change the size of the numbers in the question to be less.

Doing these things will help students learn and practice efficient calculating strategies. It will also help prevent students from becoming entrenched in using counting as their primary strategy to solving addition and subtraction.

Students who hold onto counting to add and subtract **are at mathematical risk**. Examples of counting by ones to calculate include tapping fingers, eye nods, drawing lines on paper, using the marks on a ruler or making repeated jumps of one on a number line.

Moving from Counting to Calculating

Concrete

Most manipulatives can be counted by ones.

Many students don't progress from counting by ones.

The Missing Link

Representational

Bond Blocks are a physical manipulative to support Singapore, bar-model maths.

The Missing Link to Abstract Calculation

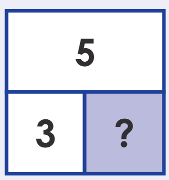
Bond Blocks cannot be counted by ones.

- Learn number bonds (facts) in a self-correcting way.
- Relate addition and subtraction using Part-Part-Whole.

Abstract

3 + 2 = 5
2 + 3 = 5
5 - 3 = 2
5 - 2 = 3

Extend with algebraic thinking.



5 - ? = 3
3 + ? = 5

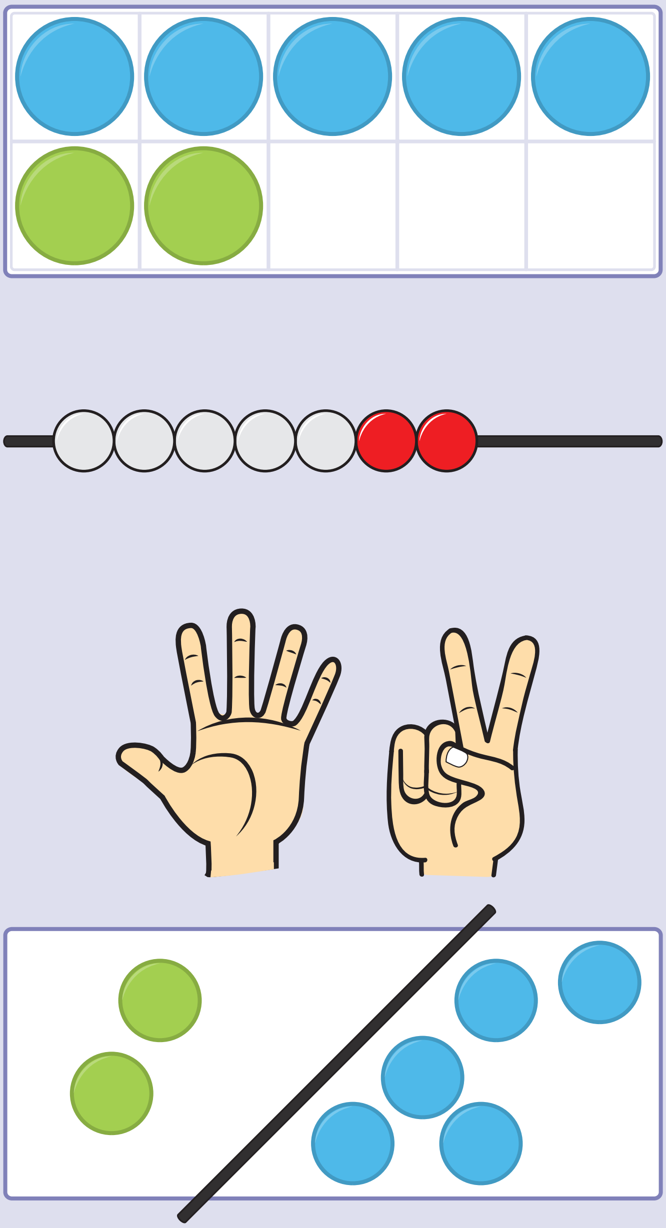
Bond Blocks fill the **missing link** to help students move from concretely **counting by ones** to abstractly **adding with numbers and symbols.**

Structuring Materials to Develop Calculation

There are many structured materials that will help students help to calculate. These include:

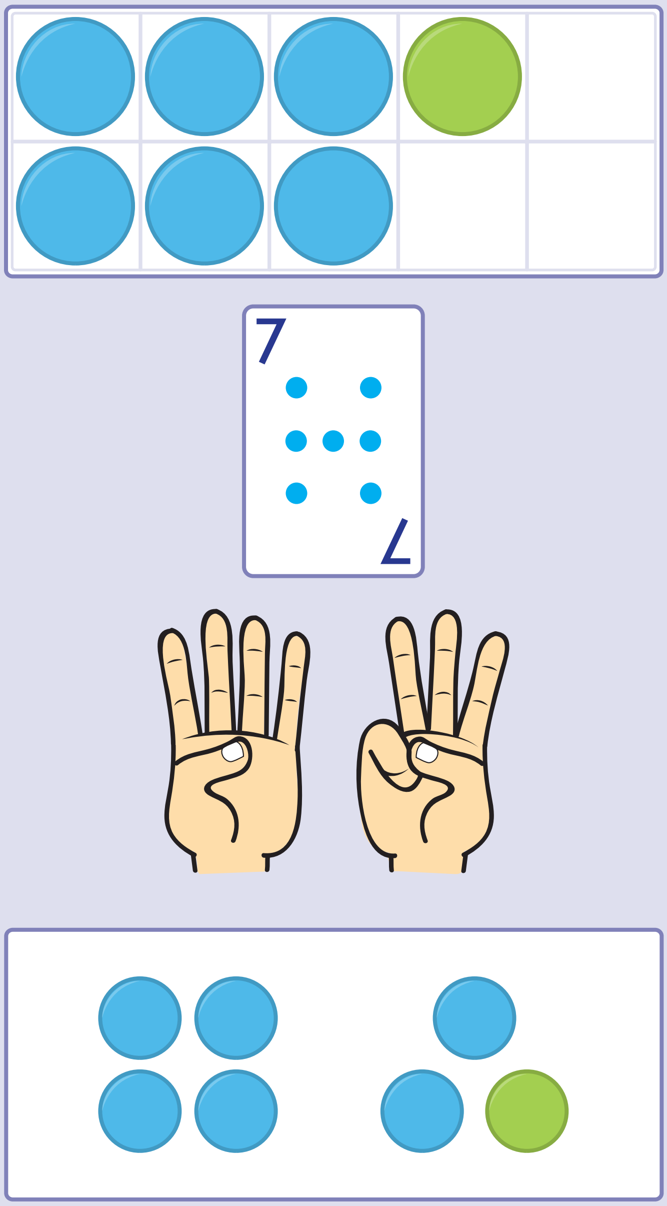
- Using ten frames to scaffold 5 plus bonds, doubles and near doubles.
- Using partition boards to separate counters into easily seen groups.
- Organising counters into standard arrangements, such as those found on playing cards and dominoes, so students identify a quantity without counting all.
- Using fingers in a subitised way instead of counting each finger by ones.

7 as a 5 plus bond



Partition board

7 using near doubles



Standard arrangements

Using counters in a non-structured way is not wrong but it teaches counting, not calculating. The Bond Block system makes the distinction between counting and calculating clear.

Subitised Use of Fingers

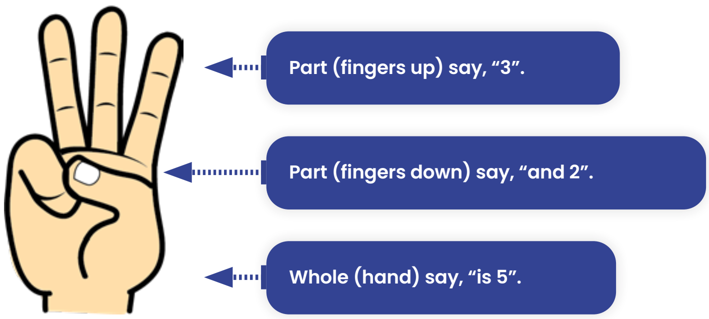
"If finger counting by ones to add and subtract is such a problem why then do fingers appear in the Bond Block Core Kit activities?"

The previous section described how using fingers to add and subtract by counting in ones can be detrimental. This section describes how fingers can be used productively, in a subitised way to develop calculation. Subistising means seeing without counting. This is different to using fingers to track counting one by one.

Using fingers in a subitised way can be the concrete step in a concrete-representational-abstract approach. The next step is to move on from fingers to the representational stage of using Bond Blocks. Finally, we want them to stop using the blocks and move onto the abstract stage of using numbers and symbols.

I can see part-part-whole.
I can see and feel "3 and 2 is 5".

$$3 + 2 = 5$$



So this means "2 and 3 is also 5".

$$2 + 3 = 5$$

I can use this to work out.

$$5 - 3 = 2$$

$$5 - 2 = 3$$

The research refers to this as "finger gnosis". For more information see:

Bisnaz, J., Fast, L., Kamawar, D., LeFevre, J., Penner-Wilger, M., Skwarchuk, S., & Smith-Chant, B. (2009). Subitizing, Finger Gnosis, and the Representation of Number. Proceedings of the Annual Meeting of the Cognitive Science Society, 31.

Systematic Subitising

Using fingers in a subitised way to add and subtract is taught systematically through the Bond Blocks Core Kit in two different ways:

- Bonds related to five. This includes **'Bonds of 5'**, **'Five Plus Bonds'**, **'Bonds of 10'**.
- Bonds related to doubles. This is **'Doubling and Halving to 10'**, then **'Doubling and Halving to 20'**.

Bonds Related to Five

Chapter 2 - Bonds of 5 Once students can make Bonds of 5 they use these to make Five Plus Bonds.	5 as 1 and 4	5 as 2 and 3	5 as 1 and 4	5 as 1 and 4
Chapter 4 - Five Plus Bonds Once students can make Five Plus Bonds they use these as Bonds of 10.	6 as 5 and 1	7 as 5 and 2	8 as 5 and 3	9 as 5 and 4
Chapter 5 - Bonds of Ten Fingers up represent one part. Fingers down represent the other part. Both hands represent the whole.	10 as 6 and 4	10 as 7 and 3	10 as 8 and 2	10 as 9 and 1

Bonds Related to Doubling and Halving

Fingers are used in a different arrangement for doubles and near doubles.

Doubles and Halves Each hand shows the same number of fingers.	2 as 1 and 1	4 as 2 and 3	6 as 3 and 3	8 as 4 and 4	10 as 5 and 5
Near Doubles One hand has one more or less than the other.	3 as 2 and 1	5 as 3 and 2	7 as 5 and 3	9 as 5 and 4	

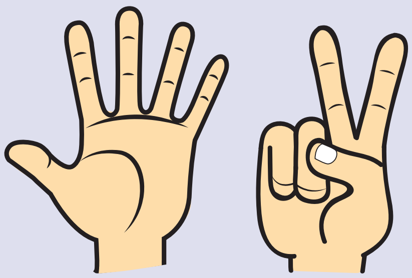
Using Fingers to Develop Calculation

Students need to trust the quantity of the whole and the partitions into which the whole is split. To move students away from finger counting by ones, students are encouraged to trust the quantity by using their fingers to subitise the whole. Subitising means seeing the quantity WITHOUT counting. This is very different from using fingers to count.

Using fingers in a subitised way reinforces;

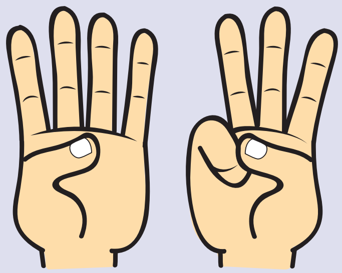
- The number bond and partitioning of the whole. For example, 7 partitioned into the bond of 5 and 2.
- Both concepts of addition and subtraction, as well as relationships between them.
- Solving subtraction by either taking away or adding on.

7 as 5 and 2

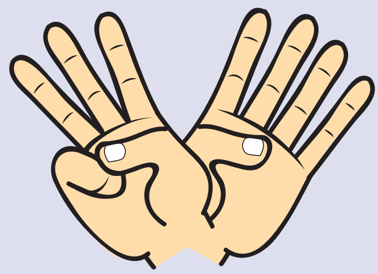


5 + 2 = 7
2 + 5 = 7
7 - 5 = 2
7 - 2 = 5

7 as 4 and 3

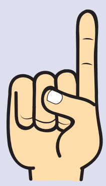


3 + 4 = 7
4 + 3 = 7
7 - 3 = 4
7 - 4 = 3

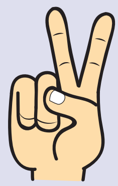


Swap hands to show the commutative property of addition.

Finger Subitising to Support 5 Plus Bonds



1



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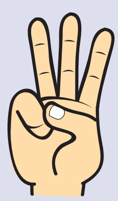

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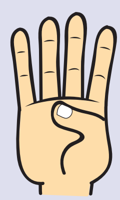

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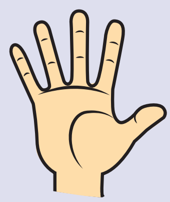

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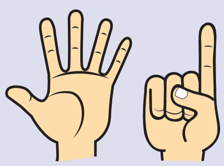
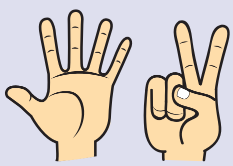
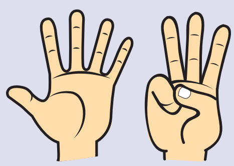


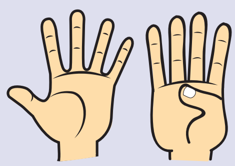
6 as 5 + 1
so
1 + 5 = 6
6 - 5 = 1
6 - 1 = 5



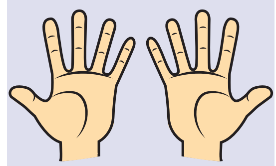
7 as 5 + 2
so
2 + 5 = 7
7 - 5 = 2
7 - 2 = 5



8 as 5 + 3
so
3 + 5 = 8
8 - 5 = 3
8 - 3 = 5



9 as 5 + 4
so
4 + 5 = 9
9 - 5 = 4
9 - 4 = 5



10 as 5 + 5
so
10 - 5 = 5

Shown with right hand dominance. The fingers on the right hand change each time.

Note: Students with fine motor difficulties may not be able to separate fingers easily or use their thumb to hold other fingers. Do not insist these students use a representation that is difficult for them. Consult their occupational therapist for advice.

Students At Mathematical Risk

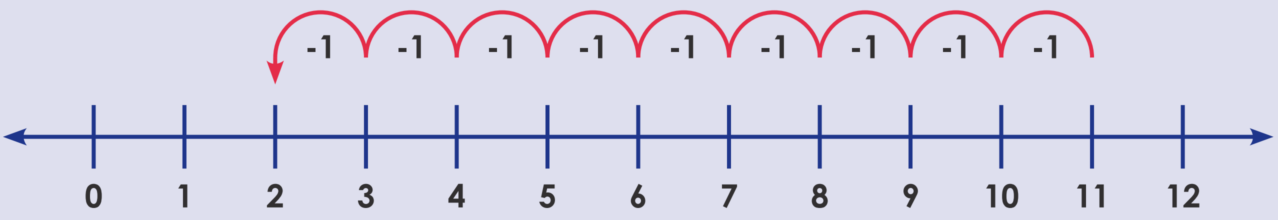
Research repeatedly highlights that students at mathematical risk or those who have a specific learning difficulty in mathematics (Dyscalculia):

1. **Lack of fluent recall of number bonds.**
2. **Lack of an understanding of the properties of addition and subtraction and relationships between them including:**
    - The commutative property of addition
    - Part-part-whole
    - The inverse relationship between addition and subtraction
3. **Lack flexible, efficient calculating strategies which use these understandings.**
4. **Persist in counting by ones to add and subtract.**

Students at mathematical risk or those who have a specific learning difficulty in mathematics (Dyscalculia) typically solve subtraction by taking away only. For example, to solve 11-9, they start at 11 and count back 9 by one.

Subtraction solved using Taking Away

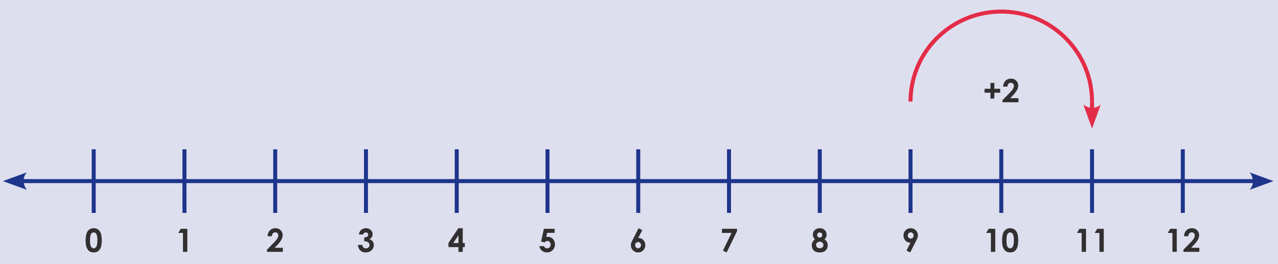
- **Inefficient counting by one**
- **Error prone**
- **Leads to dependence on counting**



This is an inefficient, incomplete understanding of subtraction that lacks the foundation for more abstract mathematics including algebra. Whereas 11-9 is more efficiently solved using 9+2.

Subtraction solved using Adding On

- **Efficient partitioning**
- **Leads to development of increasingly complex mathematics.**



Students who use counting forwards and backwards by ones, as their principle strategy to add and subtract, are at significant risk of ongoing difficulties in mathematics. Examples of counting by ones to calculate include tapping fingers, eye nods, drawing lines on paper, using the marks on a ruler or making repeated jumps of one on a number line.

Counting forwards or backwards more than three impedes the development of calculation for these reasons:

- Students who use counting as their primary strategy to solve addition and subtraction become entrenched in trusting this strategy. Because they choose counting, they do not practise new strategies enough to develop fluency.
- Effective mathematicians select efficient calculating strategies based on the size of the numbers and their situation. Counting more than three is error prone and inefficient.
- Using fingers to count add or subtract involves a double count. The student has to count forwards or backwards whilst tracking how many to add or take away. Working memory is stretched when using inefficient methods such as those involving "double counting".

Visuals support students with memory difficulties to engage in higher order mathematical thinking whilst increasing student trust and use of number bonds.

5		5		5	
0	5	1	4	2	3

Bonds of 5 Desk Visual

Bond Blocks were designed to combat these difficulties and move students from counting to calculating using addition and subtraction.

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Geary, D. C., Bailey, D. H., Littlefield, A., Wood, P., Hoard, M. K., Nugent, L. (2009). *First-grade predictors of mathematical learning disability – a latent class trajectory analysis*. Cognitive Development, 24(4), 411-429. https://doi.org/10.1016/j.cogdev.2009.10.001

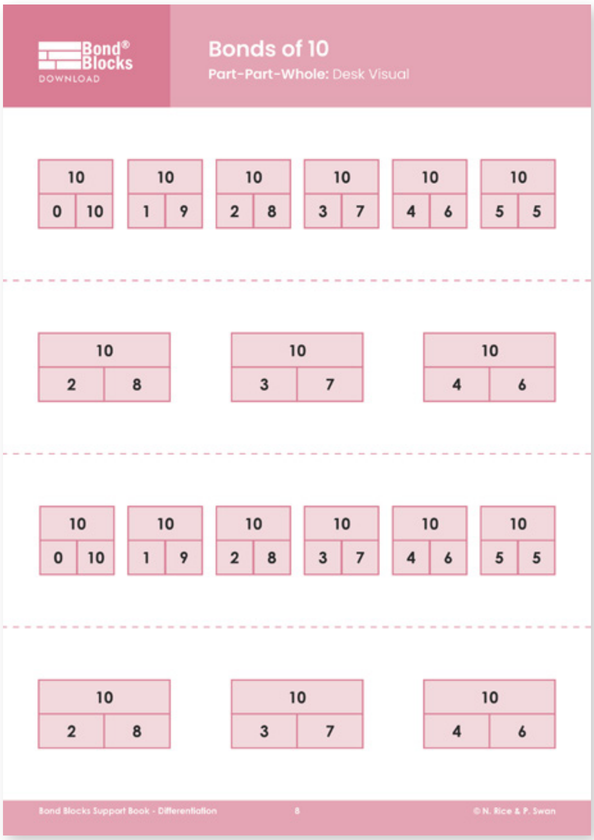
Gelman, R. & Gallistel, C. (1978). *The Child's Understanding of Number*. Cambridge, MA. Harvard University Press.

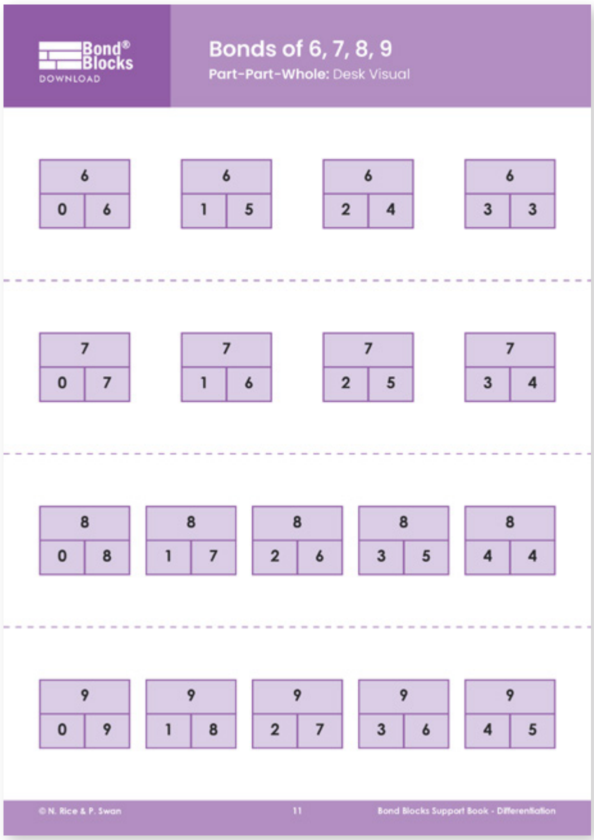
Polya, G. (1954). *How to solve it*. Princeton, NJ: Princeton University Press.

Willis, S., Devlin, W., Jacob, L., Powell, B., Tomazos, D., & Treacy, K. (2004). *First Steps in Mathematics Number. Book 2*. WA: Department of Education

Desk Visuals

Every whole in the Bond Blocks System has a downloadable Desk Visual of the related two-part bonds. These can be used to help develop fluency or as a support for students with working memory difficulties.





The Desk Visuals are ready to be cut out for individual student use.

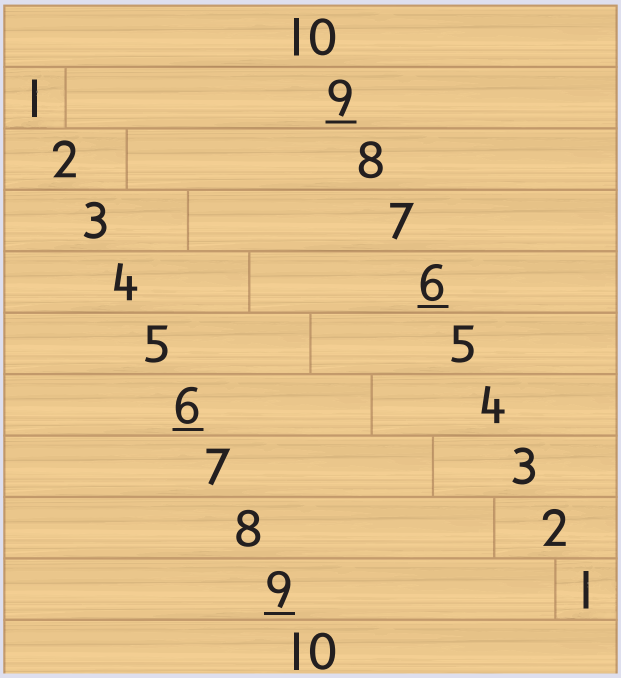
Before Using Part-Part-Whole Desk Visuals

The desk visual is used after the student has completed the **Building a Wall** activity for a specific whole to identify:

1. All of the two-part bonds and
2. The two-part bonds using the commutative property of addition.

During the **Building a Wall** activity students use Bond Blocks to construct two-part bonds and then represent them in part-part-whole diagrams. Students **must** engage in this activity prior to being given the visual aid in order to develop of robust understanding of the bonds so as they can apply these understandings to solving addition and subtraction. This is in line with the Concrete-Representational-Abstract approach underpinning the Bond Block system.

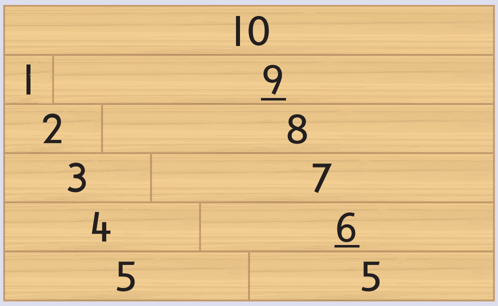
1. Building a Wall



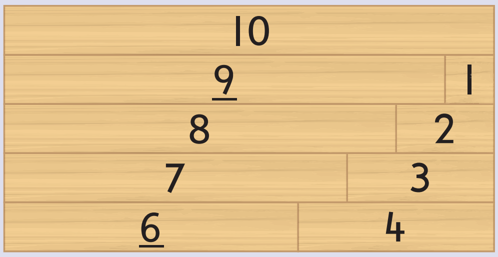
Two-Part Bonds of Ten

0 and 10
1 and 9
2 and 8
3 and 7
4 and 6
5 and 5
6 and 4
7 and 3
8 and 2
9 and 1
10 and 0

2. Applying the Commutative Property of Addition



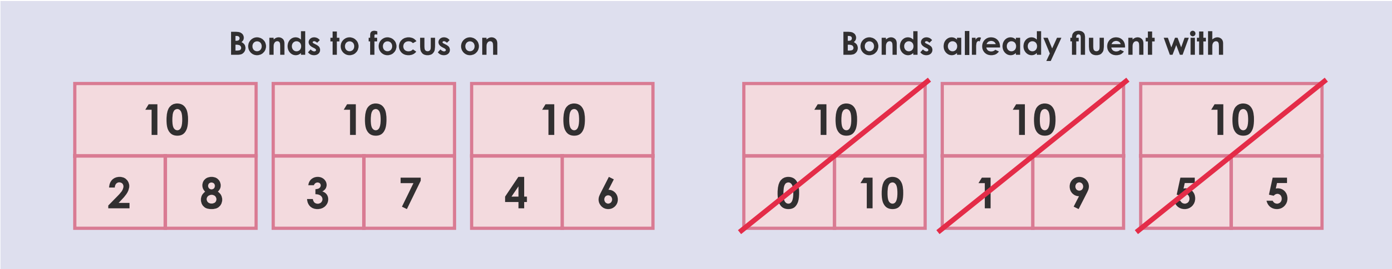
"0 and 10 is equal to 10 and 0"
"1 and 9 is equal to 9 and 1"
"2 and 8 is equal to 8 and 2"
"3 and 7 is equal to 7 and 3"
"4 and 6 is equal to 6 and 4"



Using Desk Visuals to Develop Fluency

Part-Part-Whole Desk Visuals can be cut into cards and used with students to develop fluency with two-part bonds.

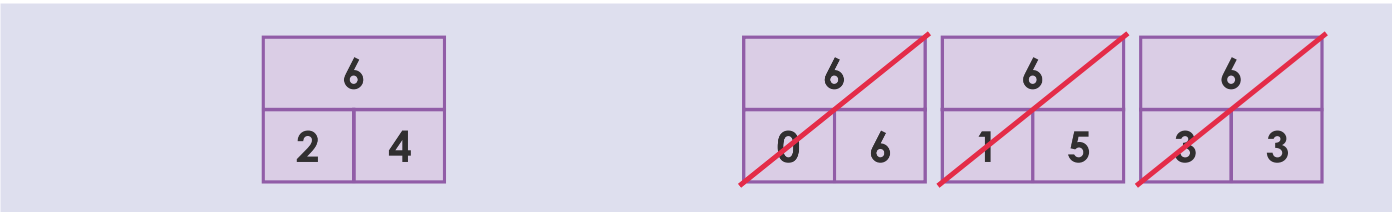
- First, choose a two-part bond visual for ONE specified whole. Cut the strip into single cards. Randomise the order of the cards. Cover different numbers on the cards to test fluency. Discard cards that the student can fluently recall.



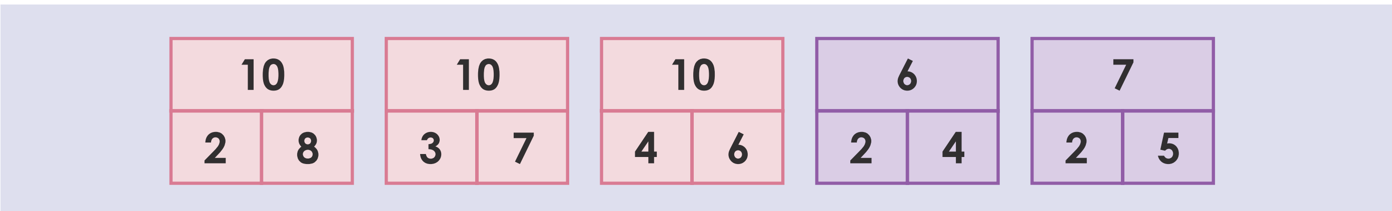
- The remaining cards can be kept and used as memory cards to practise. Cover different numbers to practise and develop fluency. When needed use fingers in a subitised way to support calculating missing numbers.

" 8 and 2 is 10"

- Once students are fluent identifying these missing numbers repeat the process for the next whole in the Bond Block sequence.



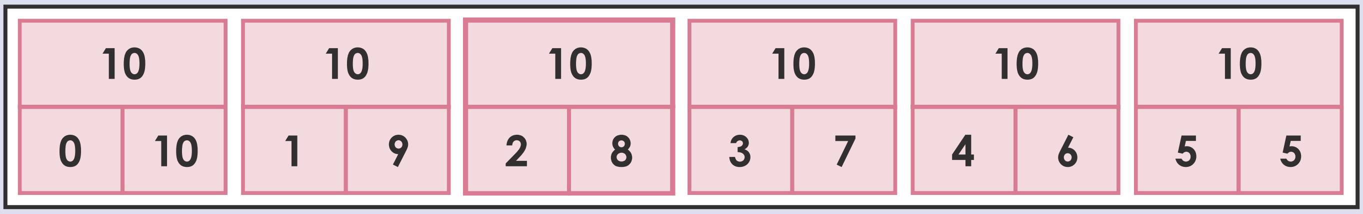
- Students can keep cards of bonds that they find difficult to remember. Students will end up with an individual collection of cards they are focusing on remembering. When students are working on remembering two-part bonds for **multiple** wholes limit the number of memory cards they are focusing on to five.



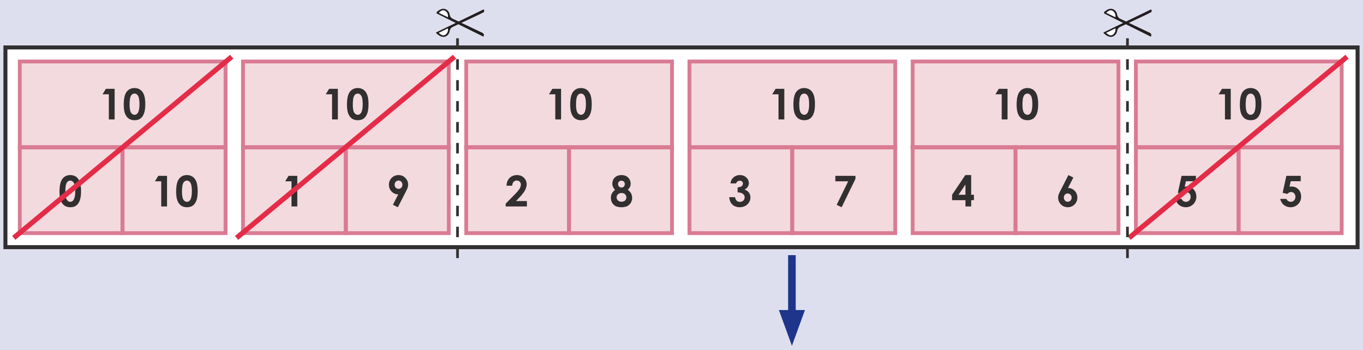
Supporting Students with Memory Difficulties

Students with memory difficulties can be supported by having constant access to relevant number bonds. Part-Part-Whole desk visuals can be used to provide this access. Having constant access to correct number bonds frees up students' working memory to engage in higher order thinking such as applying the bond to calculating strategies and solving problems.

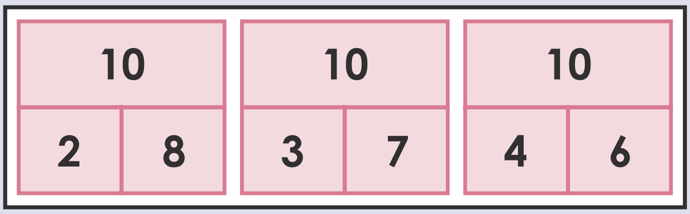
When using desk visuals be sensitive to where they are placed. Students should be involved in this decision.



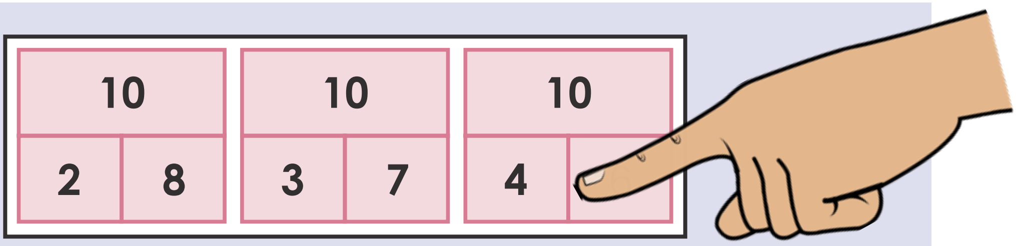
First use the two-part bond visual, for ONE specified whole, that is in counting order. The bonds are organised in counting order so they can be located easily.



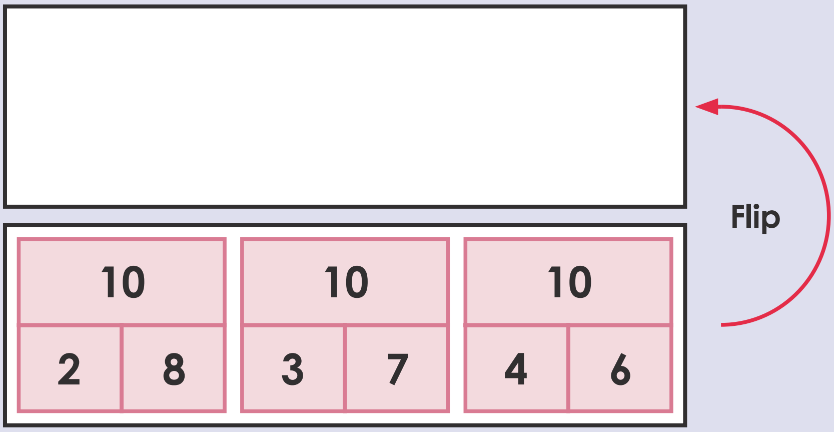
Once students develop fluency cut off bonds that are remembered. They can focus on the rest.



Use a **Look-Cover-Say-Check** approach to develop fluency.



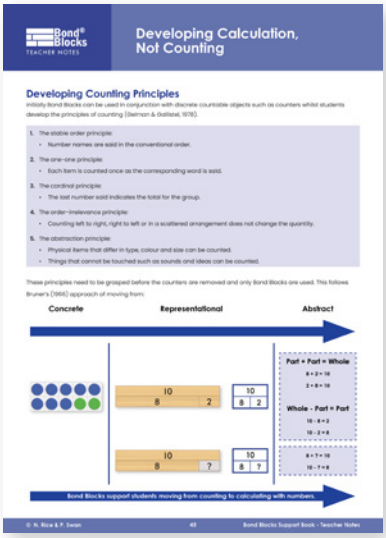
As students develop fluency recalling two-part bonds turn the desk visual face down. Encourage students to verbalise what they think the bond is before they turn it over to check.



Supporting Students with Learning Difficulties

A significant strength of Bond Blocks is that it has been designed to cater for students who experience learning difficulties. These Teacher Notes outline the features of the Bond Blocks System and Activity Boards that can support students with a range of learning difficulties.

For information to support students with Specific Learning Difficulties in Mathematics (Dyscalculia) please read the **Teacher Notes** *"Developing Calculation Not Counting"*.

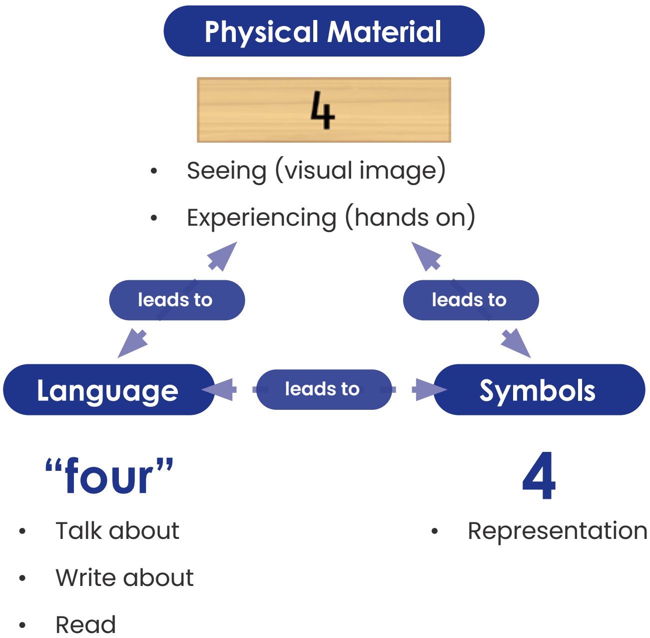


Bond Block System Features:

These features have been built into the Bond Block System to support students with learning difficulties:

Explicit Connections

Bond Blocks are explicitly linked to the appropriate mathematical language and symbols.



Video Modelled Teaching

Every activity is modelled in a video. These videos model **how to complete the activity**, but more importantly they model **connecting the mathematical manipulatives, language and symbols** to help students develop a robust understanding of the mathematical concept.

The videos are short, give clear direct instructions and are free from distracting visuals.

They can be:

- Used by teachers to introduce the activity to students.
- Used by teachers when instructing Education Assistants about the activity. Teachers introduce the activity in the first session of the week. Education Assistants can oversee students repeat the activity, the same week, to develop fluency.
- Watched independently by students. Some students like to watch the video to begin the second session of the week, as a reminder of what to do, before repeating the activity to develop fluency. Students who are anxious or find change difficult can find it helpful to watch the video independently before seeing the teacher introduce the activity for the first time that week.
- Watched by parents who may oversee fluency practice when their children bring activities home. Children with learning difficulties benefit from coordinated, consistent support through strong partnerships between home and school.

The website outlines the specific mathematics and language needed for every activity, supporting teachers to give clear, correct mathematical instruction.

Repetition of activities

The sequence and type of activities are repeated in each chapter. For example, the first activity in each chapter is building bond a wall. There are a minimal number of different types of activities. Most activities are based on Building a Wall or Tic-Tac-Toe. Developing procedural fluency with the activity enables students to focus on engaging in the mathematics.

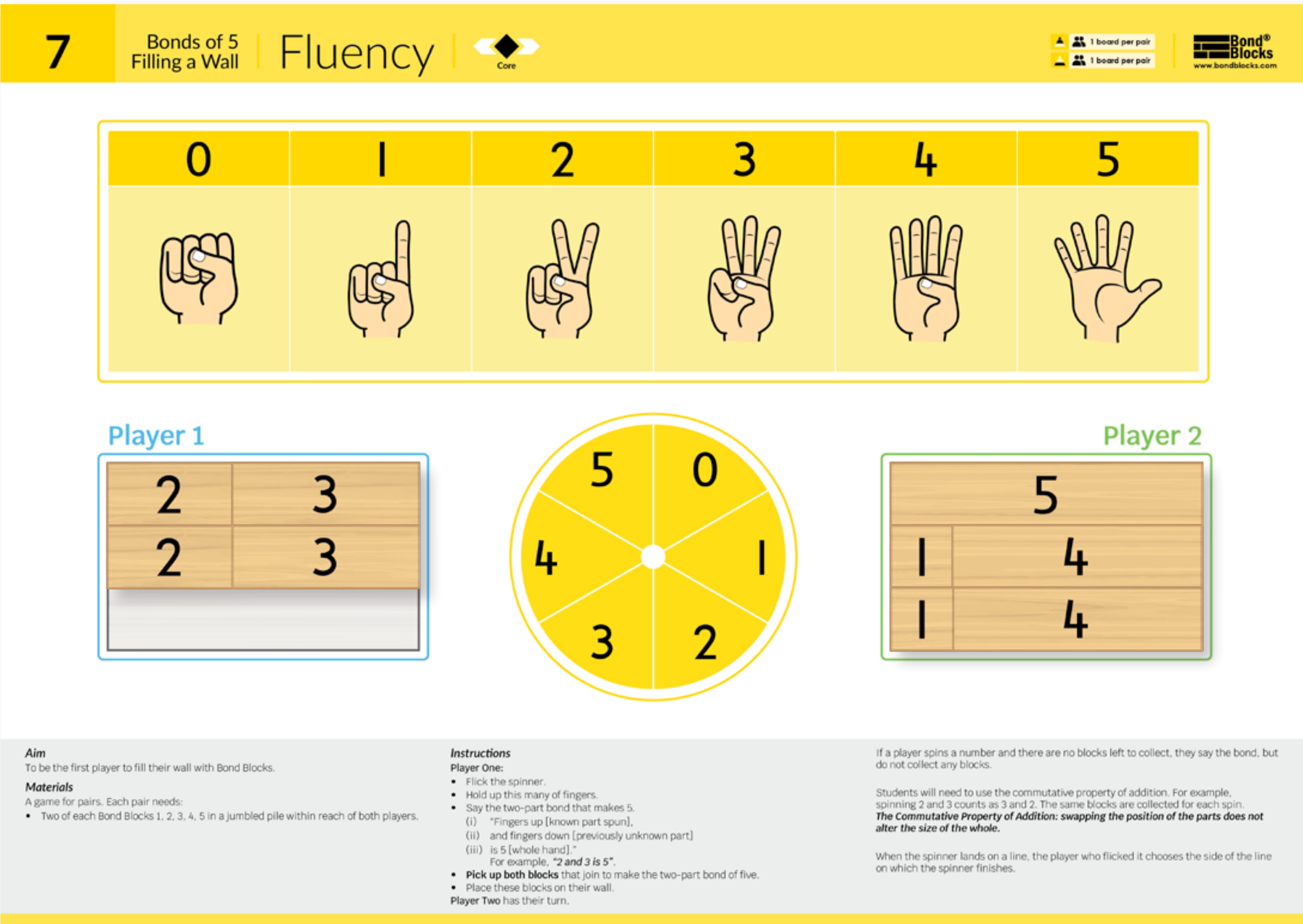
Gradual increases in difficulty

The mathematics within the Bond Block system has been broken down into carefully sequenced steps. Each activity increases in difficulty by one step and builds on the mathematics in prior activities.

Please note, that whilst activities appear the same from chapter to chapter they **change subtly** to increase the level of difficulty. These key changes are pointed out in the video and written on the activity board in bold. Please look out for these changes and highlight them to students.

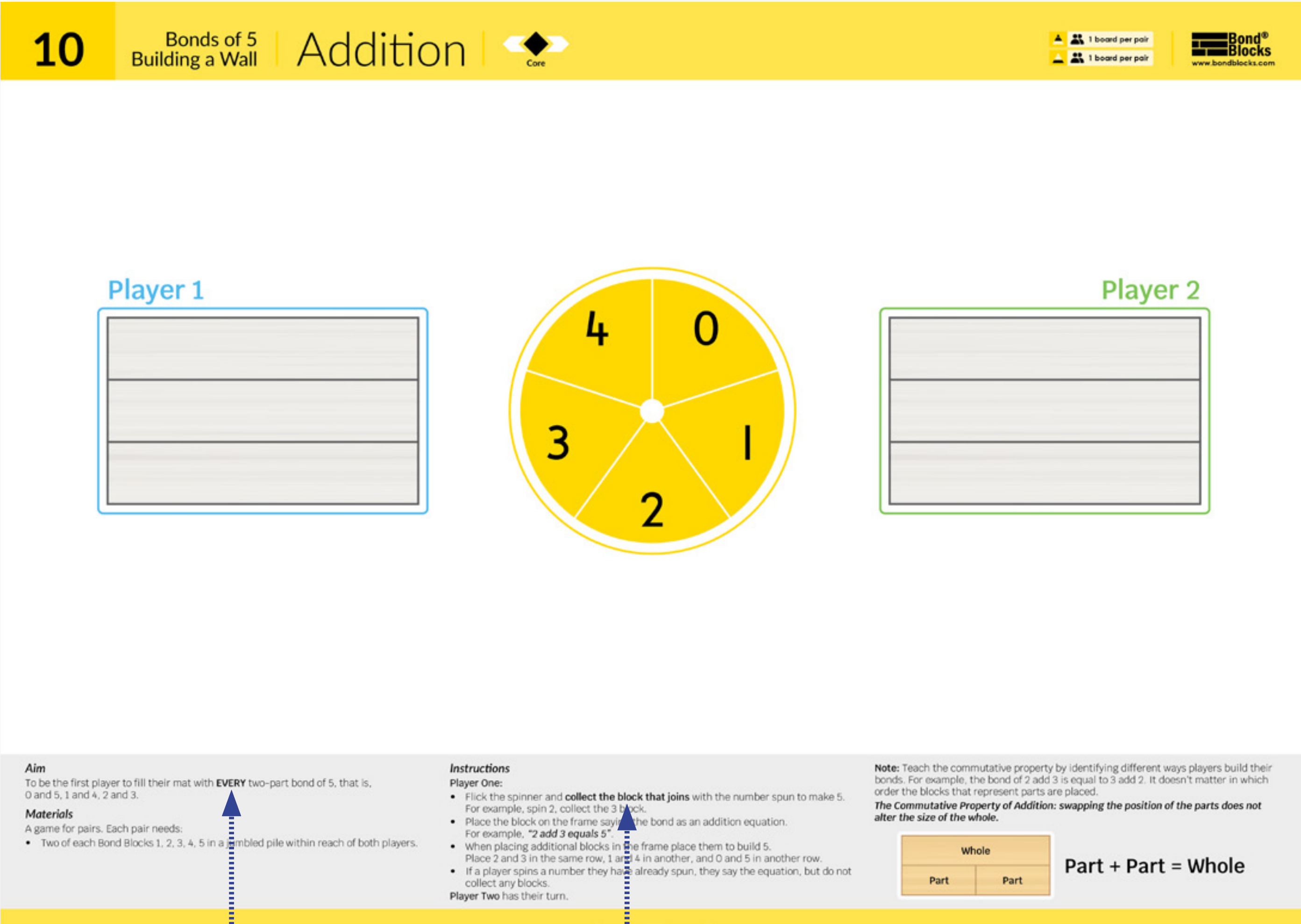
For example, the first-time students Build a Wall as a 2 Player game they:

- Have to fill their wall but can use any combination of blocks that make five. This means they can repeat bonds in their wall. Such as placing a row of 2 and 3 twice.
- They have to pick up both blocks that make the bond.



However, when this Build a Wall game appears for the second time it is slightly harder. They:

- Have to build every bond of five. That is zero and five, one and four, two and three. They are not allowed to repeat a bond such as placing two and three twice.
- Only place one block, the answer to the equation, because this is the emphasis of the addition activity. Whereas in the earlier game they had to place both blocks because the emphasis was on the two-part bond.

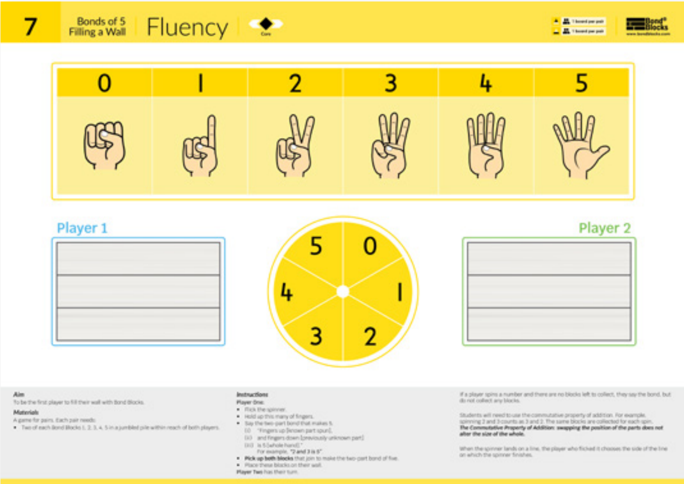


To be the first player to fill their mat with **EVERY** two-part bond of 5, that is, 0 and 5, 1 and 4, 2 and 3.

Flick the spinner and **collect the block that joins** with the number spun to make 5. For example, spin 2, collect the 3 block.

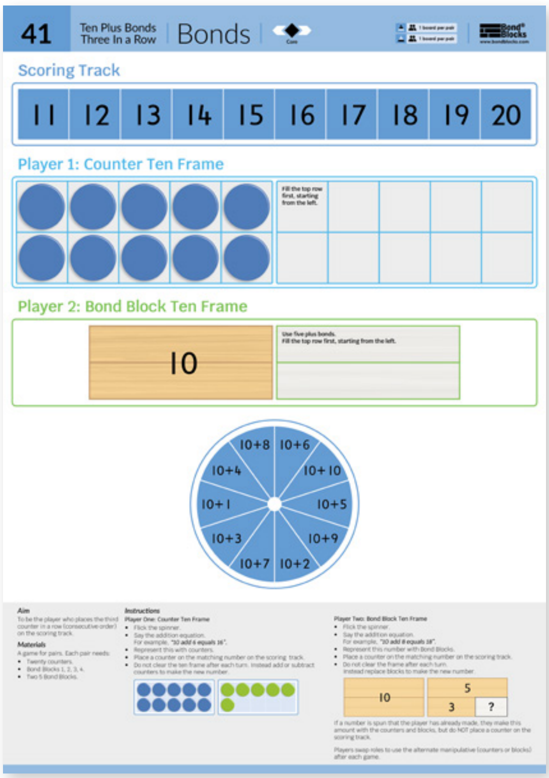
A multisensory approach.

Activities involving doing, seeing, saying, and hearing. Students do the activity with their blocks hands, see the visual representation, say the bond aloud and hear others say it. Actions and statements to be said are specifically *stated on the activity boards in **bolded italics***. The aim is to increase neurological connections, understanding and memory retention.

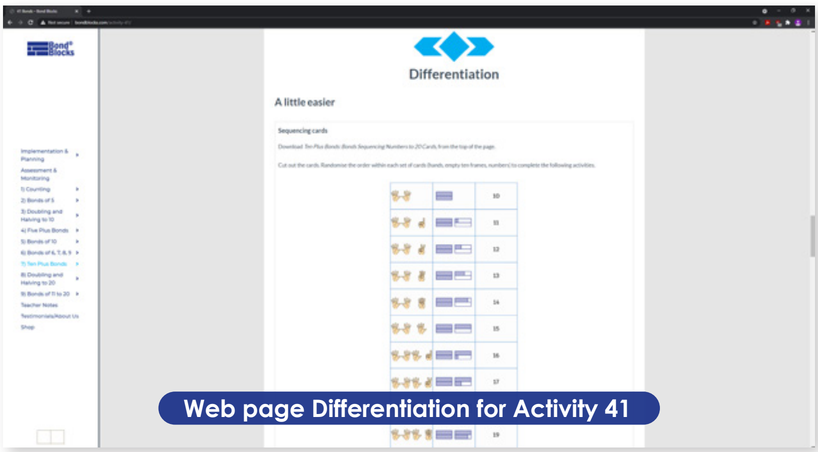


Additional support activities.

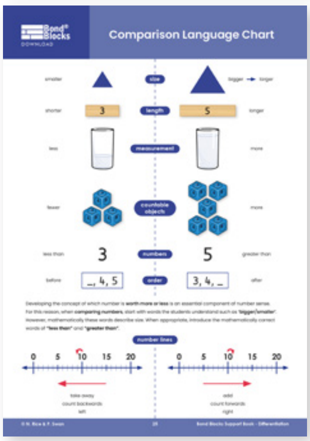
Every activity is differentiated. See the Bond Block's activity web page for adjustment suggestions and ready-to-go download activities to make the activity a little easier.

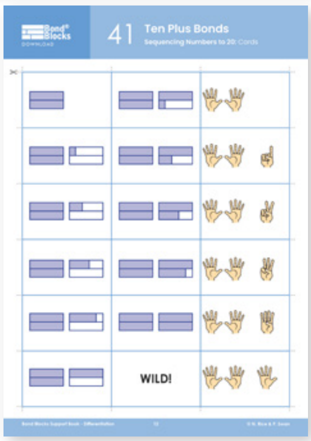


Activity 41 additional support activities.



Web page Differentiation for Activity 41

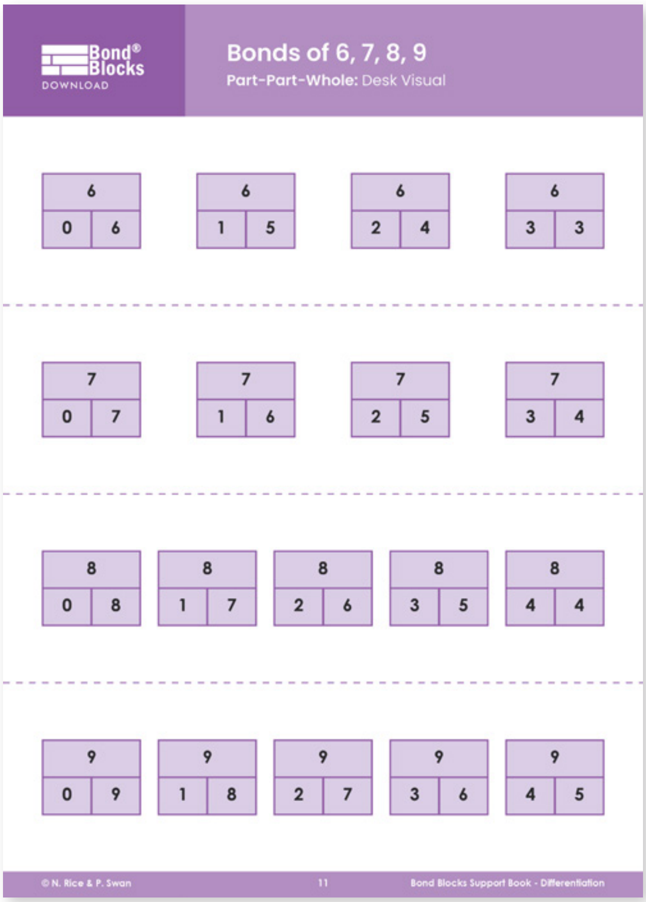




Downloads for Activity 41

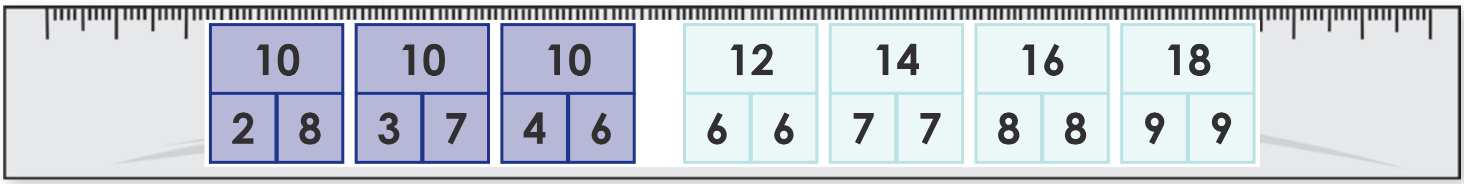
Supportive of students with memory difficulties.

Students who have difficulty remembering bonds can be supported by downloading part-part-whole visuals. Students use these when playing games. Using the visual support frees the student's working memory and allows them to engage in the higher order mathematic thinking, such a developing calculation strategies and problem solving.



Desk Visual Download for Two-Part Bonds of 6, 7, 8, 9

Small sized, part-part-whole visual strip for difficult to remember bonds of 10 and doubles. Download and tape this to the back of a ruler for discrete use by older or self-conscious students.



For more information about how to use desk visuals read the **Teacher Notes** *"Using Part-Part-Whole Desk Visuals".*

Activity Board Features

These features have been built into the Bond Block Activity Boards to support students with learning difficulties:

Free of visual distraction

This supports students with visual discrimination difficulties. The aesthetics of the game boards have been designed to be sensitive of older students using them for Wave 3 intervention.

Played in pairs

This increases the amount of time spend practising the skill and reduces the amount of time available for the student to become distracted.

Embedded with student accountability measures for:

i) correct practice of mathematics.
Bond blocks are self-checking. Frames on the boards ensure correct placement of blocks.

ii) correct use of mathematical language.
The correct mathematical language to be used is written on the activity board.

iii) students' cognitive participation.
Accountability measures are built into games. Each player has a role. For example, if the watching player identifies an incorrect calculation they move forward 3 spaces and the player in error moves back 2 spaces.

Always orientated correctly

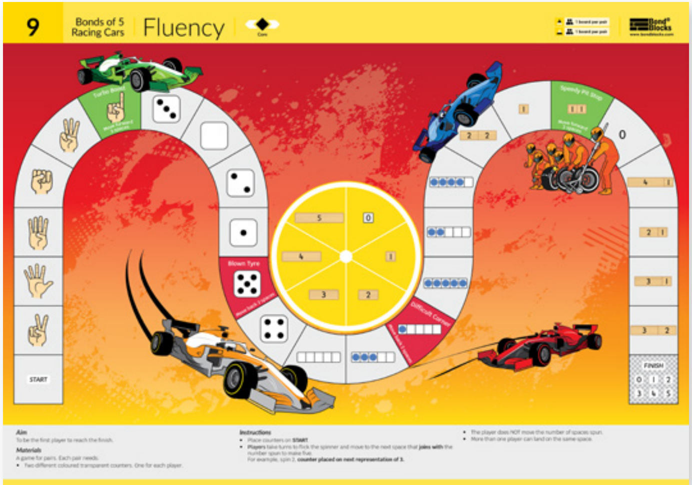
- Students engage in activities students sitting side by side.
- The A3 game board and blocks are positioned directly in front of each student.
- Player One is always coloured blue and positioned on the left. Player Two is green and on the right.

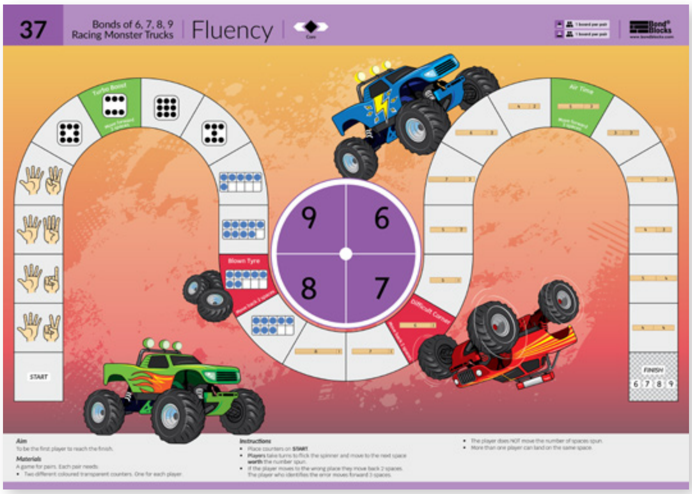
These measures help to reduce visual tracking and physical coordination difficulties.

Sitting side by side is non-confrontational for students with autism spectrum disorders.

Consistent Features of Activity Boards

- **Player One** is always blue, sits on the left and goes first. **Player Two** is always green, sits on the right and goes second. The boards and counters are colour coded blue and green.
- On track-based activity boards landing on a **green space** means move forward 3 spaces. Landing on a **red space** means move back 2 spaces. This supports students with reading difficulties.





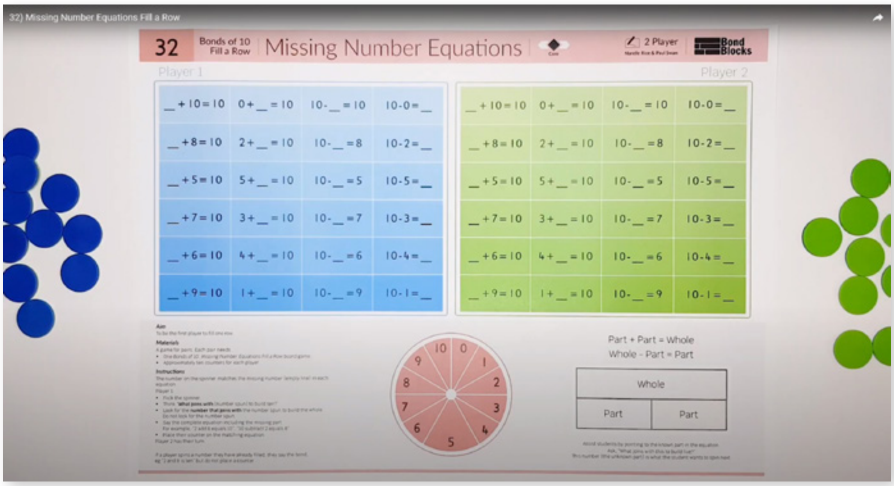
Used with minimal equipment.

Game boards are used with spinners because they facilitate efficient set up, are quite and do not contain loose pieces to distract or fall on the floor.

When activities or games require a concrete representation counters are used.

- Counters with a 25 millimetre diameter were chosen because they are easier to manipulate. The counters are blue and green to match the blue and green player colours.

Player 1
Is always blue,
positioned on this
side.



Player 2
Is always green,
positioned on this
side.

- However, students with attentional difficulties can find using counters distracting. Students with coordination difficulties can find manipulating counters difficult. Frustration can occur when counters are knocked or moved. Two ways to accommodate for this difficulty are:
  - place a small piece of sticky tack under each counter. The sticky tack works well with the plastic A3 sleeves in which the boards are placed.
  - use one dry erase marker/pair of students instead of counters. Students record moves on track games by writing an initial and draw tally marks in scoring tables. Having one marker/pair reduces opportunities for students to use the marker inappropriately.

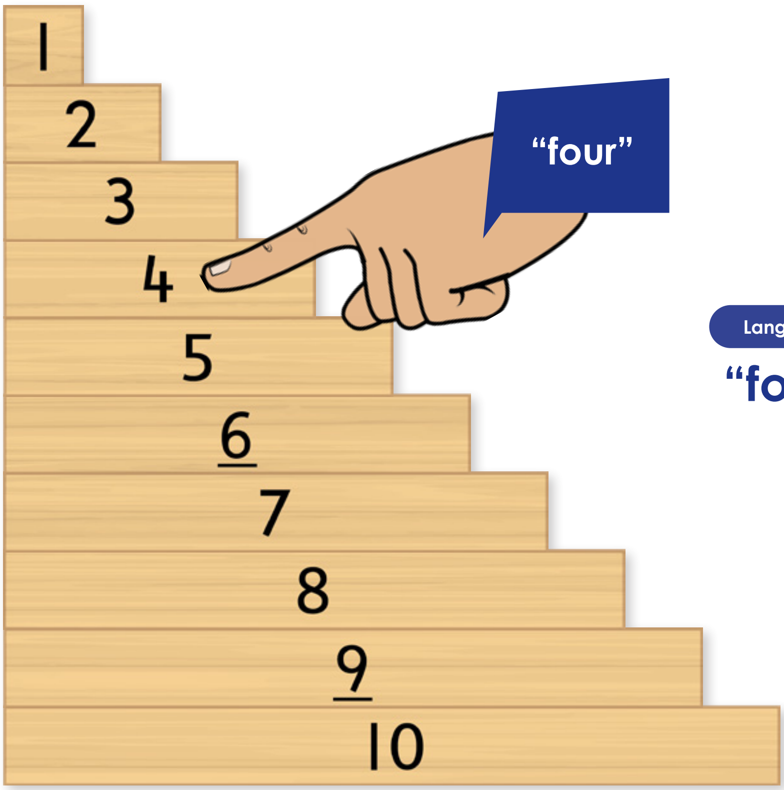
Instructions About Mathematical Language

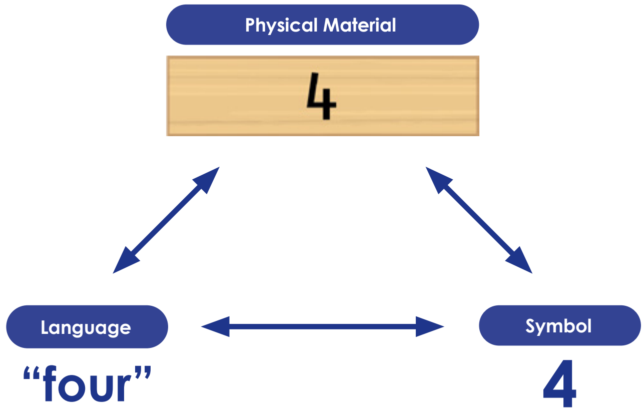
Mathematical Connections

Mathematical understanding is strengthened when connections are made between the physical materials, mathematical language and symbols (written numerals).

For this reason it is important that as students say the specified mathematics they touch the written numeral on the block, as they say the number word. Check students touch next to the numeral. They should not touch at the end of the block or cover the numeral with their finger.

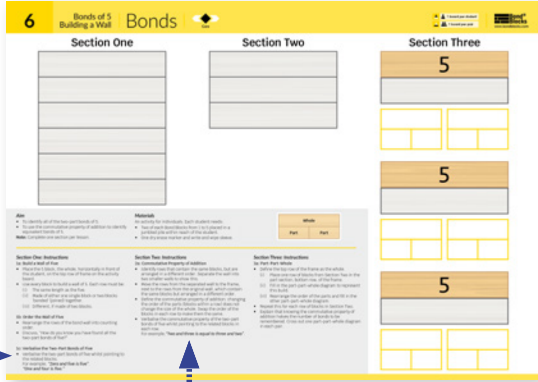
Touching next to the numeral as the number word is spoken helps students actively focus on mathematics, rather than passively echoing the choral verbalisation of bonds.





Doing this helps students connect the physical material with the language and symbols.

Verbalisation Activities



1c: Verbalise the Two-Part Bonds of Five

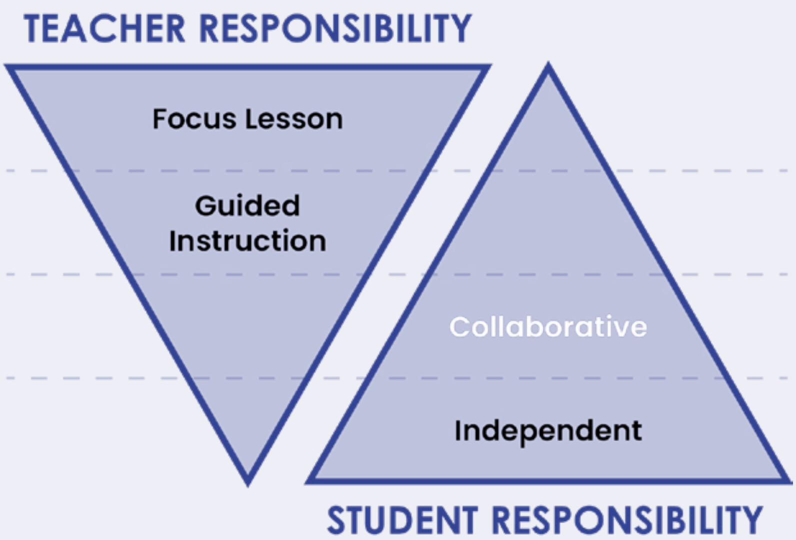
- Verbalise the two-part bonds of five whilst pointing to the related blocks. For example, *"Zero and five is five". "One and four is five."*

- Verbalise the commutative property of the two-part bonds of five whilst pointing to the related blocks in each row. For example, *"Two and three is equal to three and two".*

During One Player activities students are often required to say bonds whilst pointing to the related blocks.

This type of activity can be effectively taught using Fischer and Frey's (2008) model of gradual release of responsibility.

- **I Do:** The class watches the activity video.
- **We Do – "Say it with me":** The teacher uses the magnetic blocks on the board. The teacher and students say the bonds at the same time whilst the teacher points to the magnetic blocks on the board. The students only say the bonds, they do not touch blocks at this point.
- **We Do – "Do it with me":** The students say the bonds whilst touching the blocks on their desk at the same time as the teacher who is modelling at the front using the magnetic blocks on the board.
- **You Do:** In pairs the students take turns to say the bonds touching the blocks on their desk. The teacher walks around checking this is being done correctly.



A Model for Success for All Students

Fisher, D., & Frey, N. (2008). Better Learning Through Structured Teaching: A Framework for the Gradual Release of Responsibility. Alexandria, VA: Association for Supervision and Curriculum Development.

Fisher, D., & Frey, N. (2008). Better Learning Through Structured Teaching: A Framework for the Gradual Release of Responsibility. Alexandria, VA: Association for Supervision and Curriculum Development.

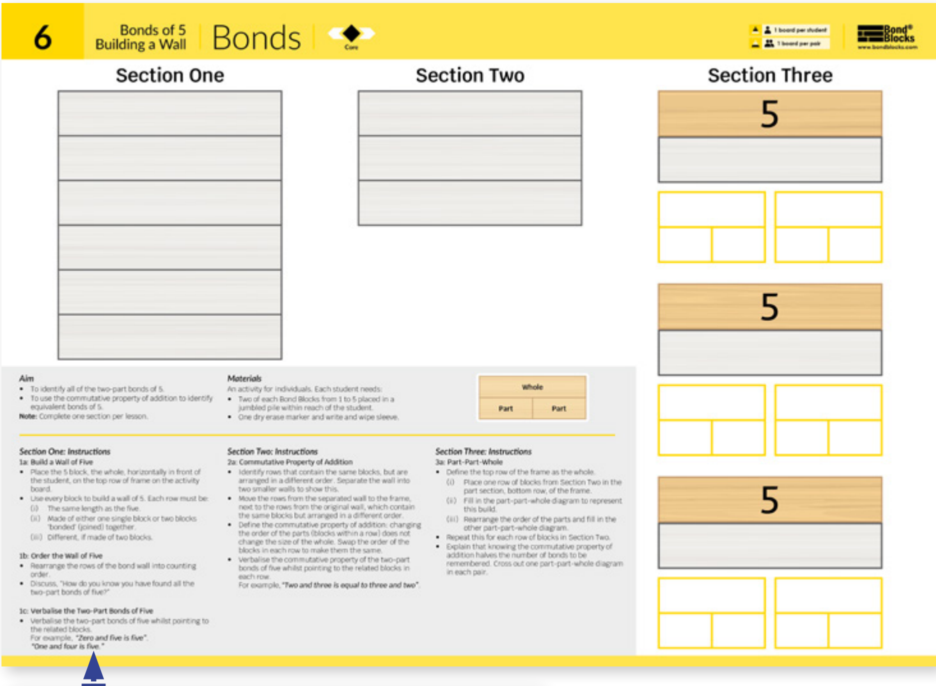
The Specified Language

The use of language throughout the Core Kit is consistent and developmental. It has been carefully considered to reflect the correct mathematical concept. Please use the specified language. The specified language for bonds, addition, equivalents and subtraction is outlined below.

Bonds

Bond activities use the terms "and" and "is". For example, "two **and** three **is** five".

Please try not to say "two **add** three is five" in the bond activities. The reason the word "add" is not used during Bond activities is that the goal is for students to remember the bond as a fact. Then apply this to addition **and subtraction**. If students first learn the bond using the word "add" they tend to remember the addition equation, not the bond, and then have difficulty applying this subtraction.



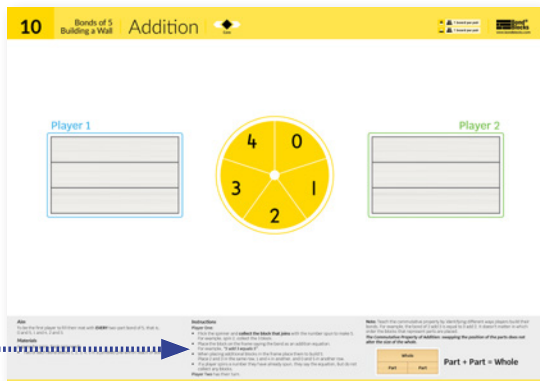
"Zero and five is five. One and four is five."

Students practise the bonds in the Fluency Activities that follow the Bond activity. Initial Fluency Activities in each chapter instruct students to say the bonds. Fluency activities that occur later in the chapter do not. This is because the students have had sufficient time practising saying the bond and are now focusing on identifying it fluently.

Addition

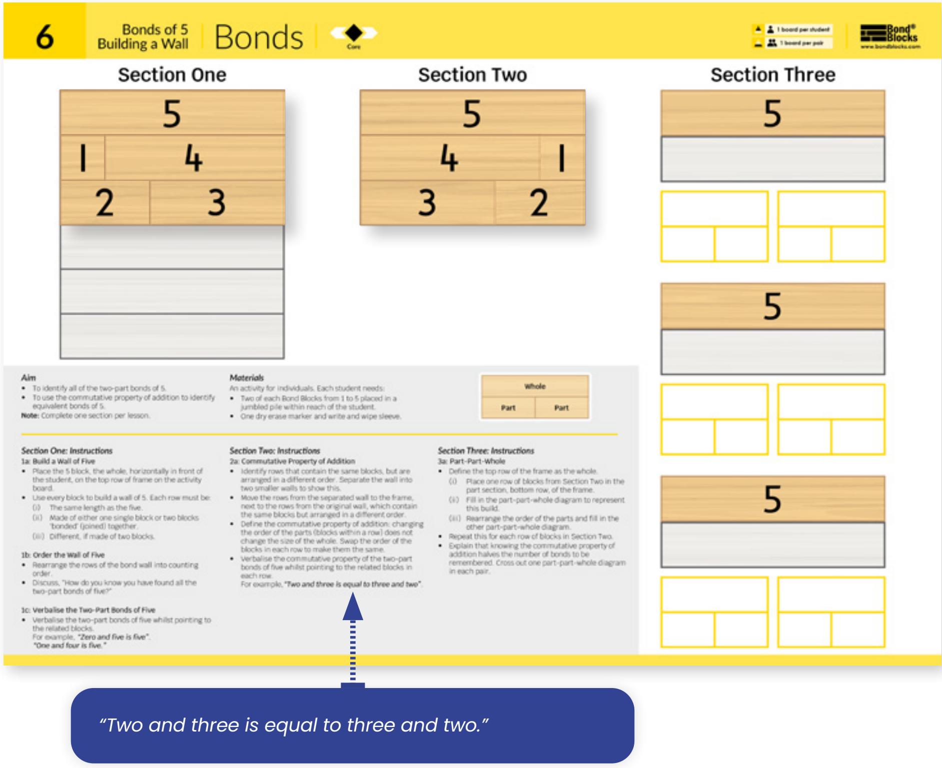
Later the bond is applied to addition. At this point the language changes from "and" to "add", and "is" becomes "equal". For example, "two **add** three **equals** five".

- Place the block on the frame saying the bond as an addition equation. For example, *"2 add 3 equals 5"*.



Equivalence

When describing equivalence for the commutative property of addition the phrase "is equal to" is used. For example, "two and three **is equal to** three and two".

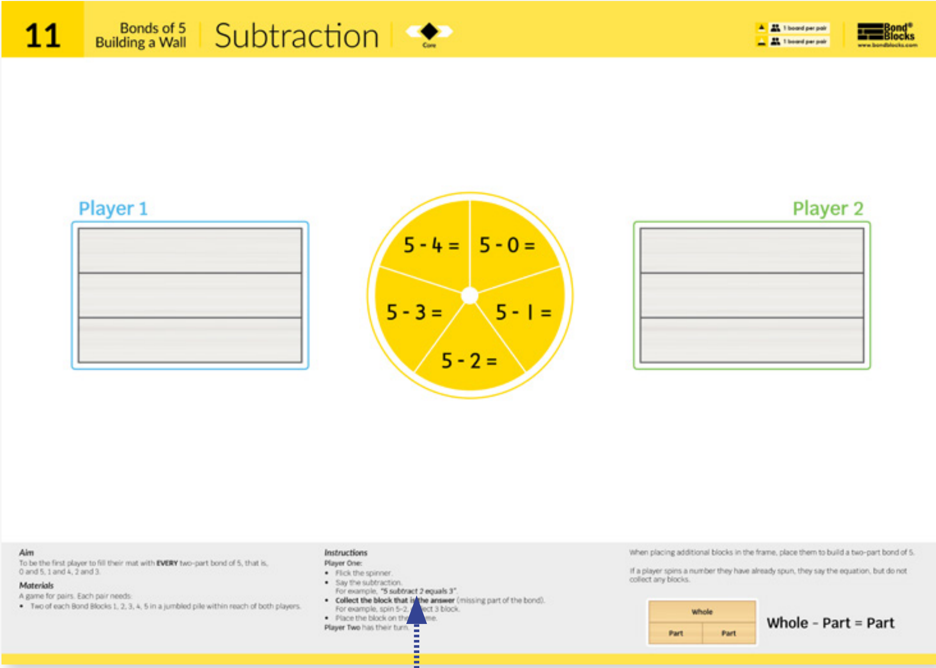


Please try not to say "is the same as" during equivalence activities.
However, **they are not "the same as"** because the blocks (representing the parts) are in a different order.

Two and three **"is equivalent to"** three and two because they are the same length (representing the whole).

Subtraction

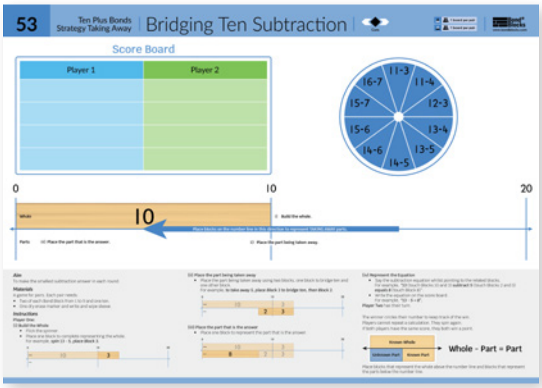
When applying the bond to subtraction the word “subtract” is used. For example, “Five **subtract** two equals three”.



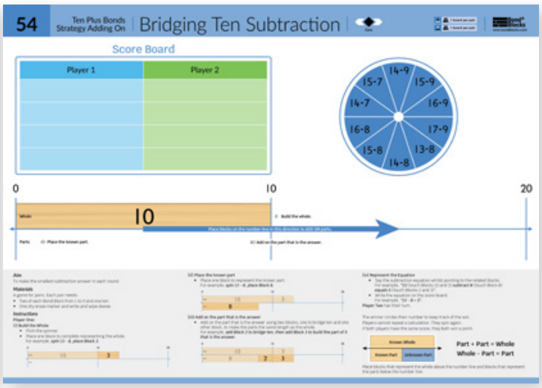
“5 subtract 2 equals 3.”

Please try not to say “take” or “take away” during these activities. This is for several reasons:

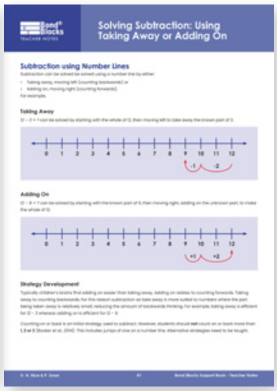
- The name of the operation is “subtraction” not “take-away-ion”.
- The word “subtraction” is taught less frequently than “take away” so needs to be prioritised.
- Subtraction can be solved by taking away, but it can also be solved by adding on. For more information explaining this please see the Teacher Note **‘Solving Subtraction Using Taking Away or Adding On’**. Bond Block Activities specify when subtraction is to be solved using adding on or taking away.



53: Ten Plus Bonds - Bridging Ten Subtraction - Strategy Taking Away



54: Ten Plus Bonds - Bridging Ten Subtraction - Strategy Adding On



Solving Subtraction Using: Taking Away or Adding On

- Typically, students at risk in mathematics only understand and solve subtraction as taking away. This impedes their progress in subtraction. Bond Blocks focuses on helping students understand the connections between addition and subtraction and develops a range of efficient strategies to add and subtract. For more information about students at risk in mathematics see the Teacher Note **‘Developing Calculation, Not Counting’**.


6
7
8
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