Part-Part-Whole from Cubes to the Bar Model

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Part-Part-Whole is used to help students develop additive thinking, connecting addition and subtraction. This article will show you the progression of Part-Part-Whole from using cubes, to a bar model and the equation.

Part-Part-Whole Using Cubes

When using **discrete materials** to represent addition and subtraction, encourage students to arrange these so the quantity can be seen without counting. This encourages students to develop partitioning-based strategies, instead of relying on counting by ones.

Addition

- When adding, **one part** is collected and **one part** is added, then the manipulatives are combined to make one **whole**. **Part + Part = Whole.** This is demonstrated in *figure 1*.

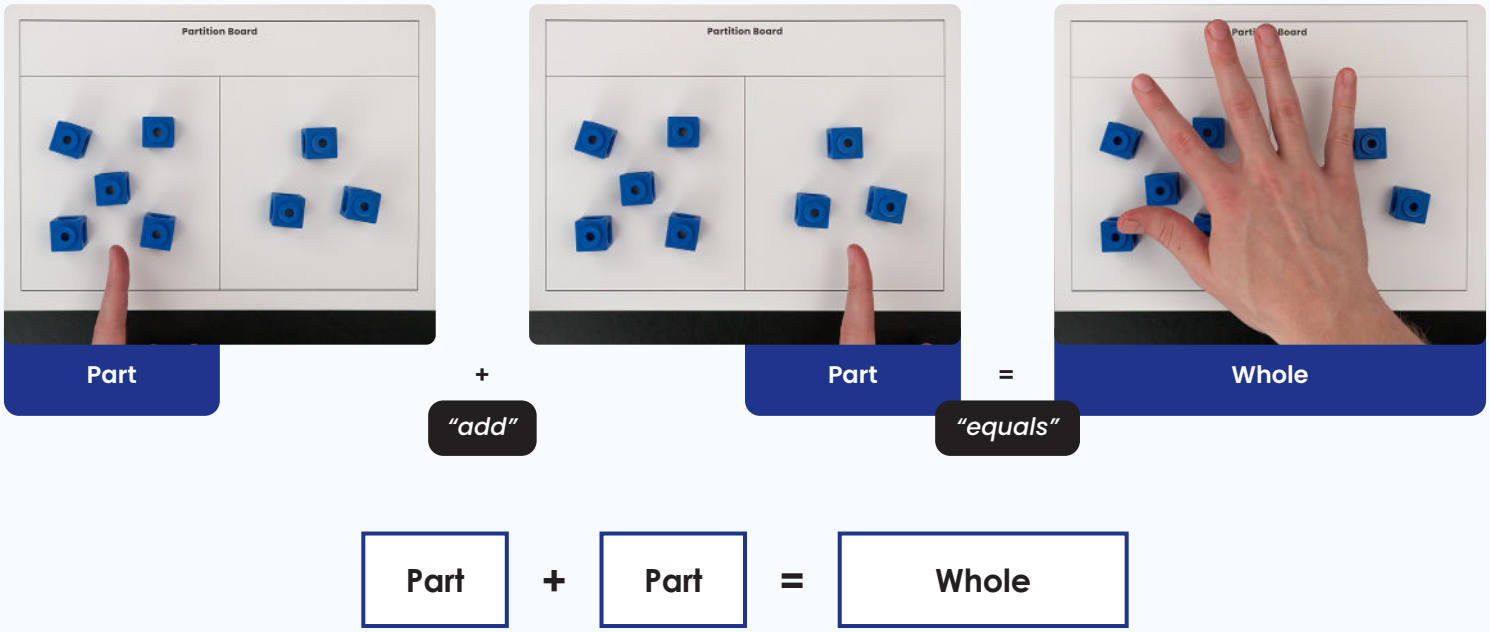


Figure 1. Part + Part = Whole

Students can use what they know to work out what they don't know yet.

For example, in *figure 2*, the student made 5 and 3 by starting with the fact they knew 4 and 4.

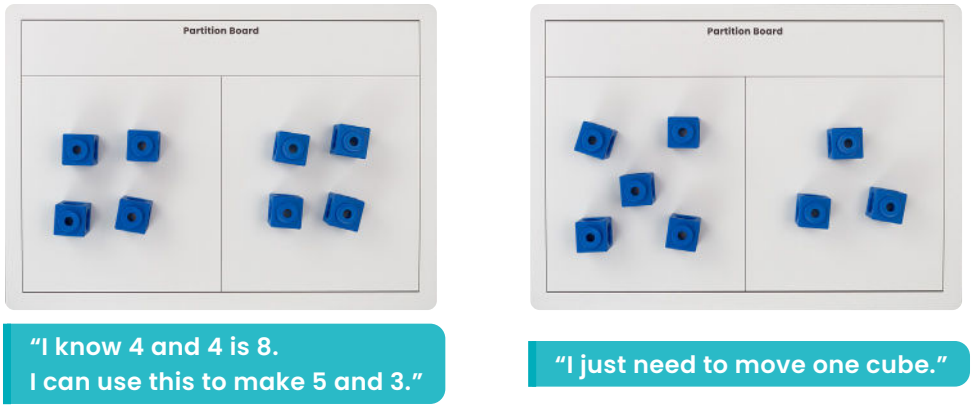


Figure 2. Using known facts to derive unknown facts

Commutative Property

a + b = b + a
Changing the order of the parts in addition does not change the size of the whole.

The board can be turned 180 degrees to show the **commutative property of addition**.

Figure 3 shows how 5 + 3 = 3 + 5.

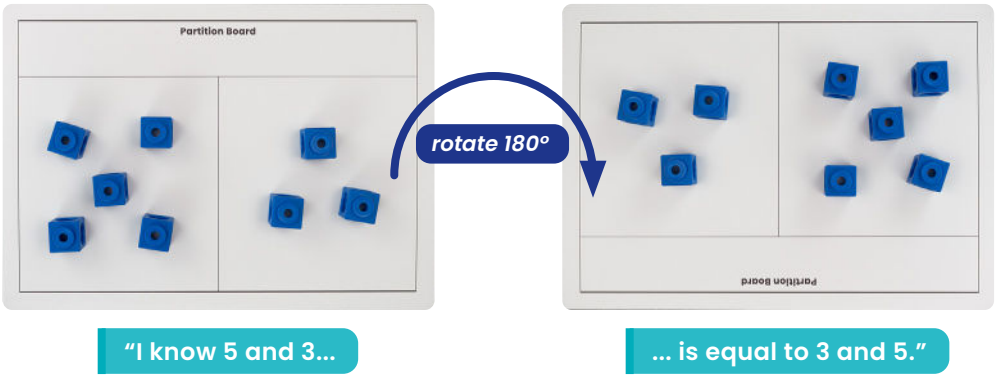


Figure 3. Using the commutative property to derive another fact

Subtraction Using Cubes

Addition is related to subtraction. This is known as the inverse property.

- When subtracting, the **whole** set of manipulatives is collected, **one part** of this set is separated, leaving the **other part**. **Whole – Part = Part.** See *figure 4*.

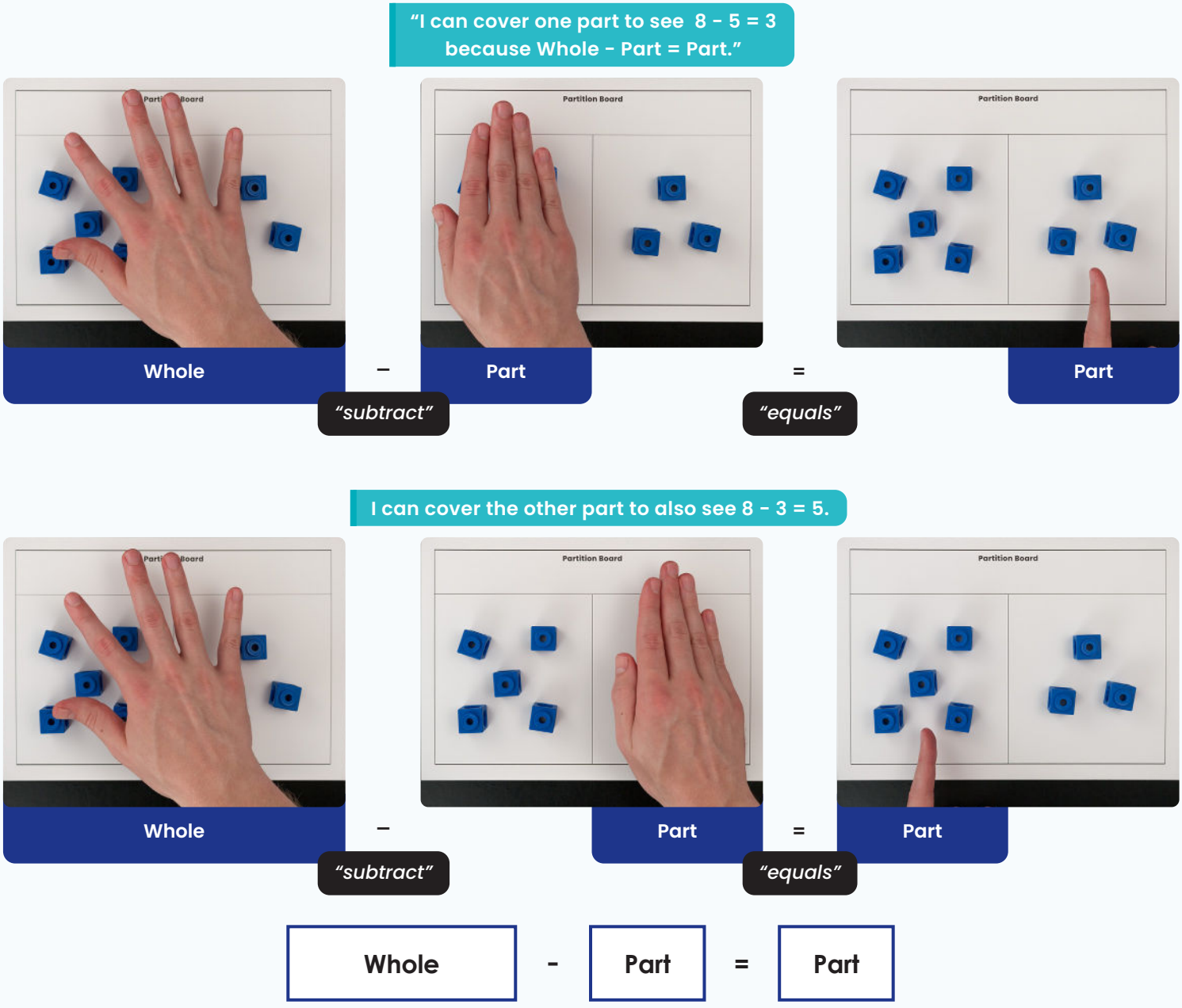


Figure 4. Whole - Part = Part

Part-Part-Whole Using a Length-Based Manipulative

A transition may be made from using countable objects, such as cubes, to a length-based model, such as a **bar model**. This can assist students to move from counting based strategies to partitioning strategies and recall.

Figure 5 shows a length-based manipulative called Bond Blocks. The blocks do not have individually scored lines that can be counted. They are a **representational manipulative** of the bar model. The length of the block and number represents the number of cubes. How the blocks are positioned and moved represents the operations of addition and subtraction.

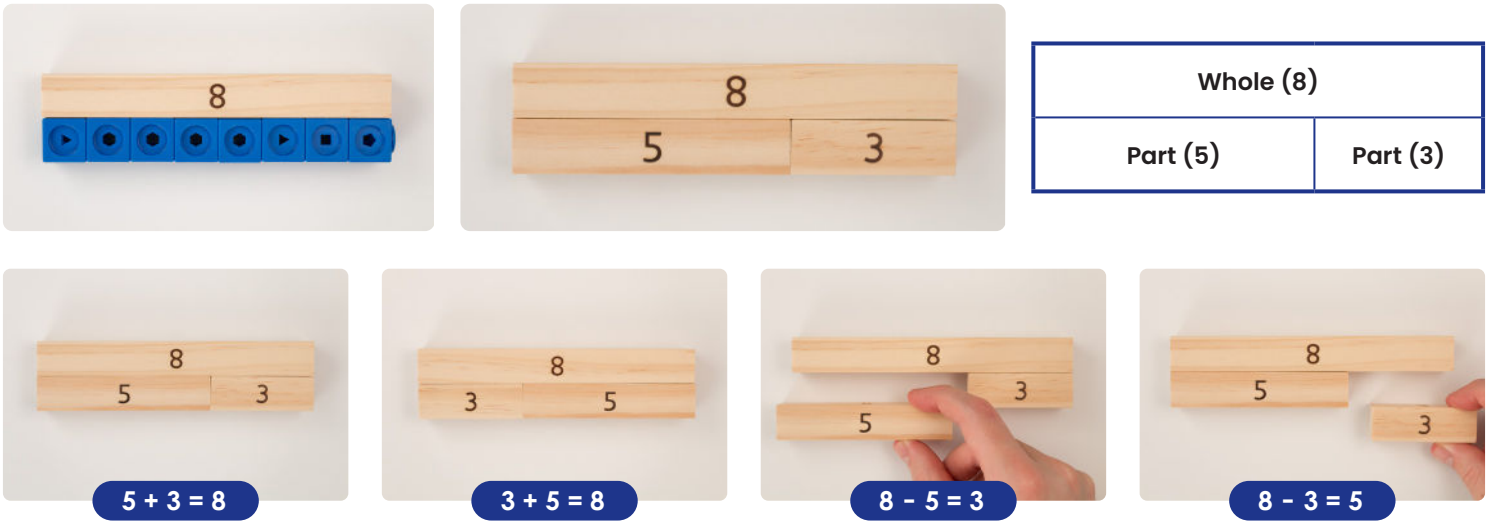


Figure 5. Using Bond Blocks to represent Part-Part-Whole

The advantage of a length-based manipulative is that they are self-checking. Students work mathematically using estimation and the guess, check and improve strategy to find the correct length block (see *figure 6*). Length-based manipulatives help students identify and practise learning the correct bonds.

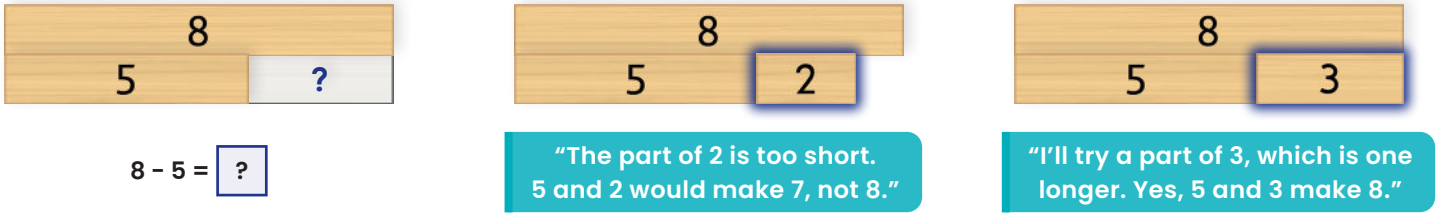


Figure 6. Length-based manipulatives are self-checking

Students can use length-based manipulatives to relate what they know to what they don't know yet. See *figure 7*.

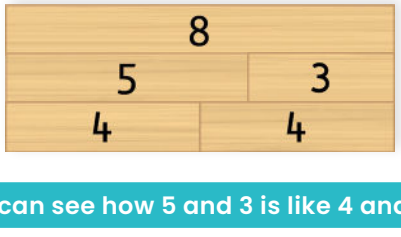


Figure 7. Relating known facts to derive unknown facts

Part-Part-Whole Using a Diagrammatic Representation

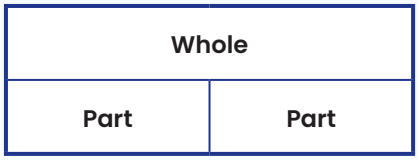


Figure 8. The bar model for *two-part* bonds

The bar model is one diagrammatic model used for addition and subtraction. Australian Curriculum documents use the diagram in *figure 8* to represent parts and wholes.

The unknown needs to be placed in different positions for different questions, as per figure 9 and 10.
This allows for links to be made to algebraic thinking.

Part Unknown

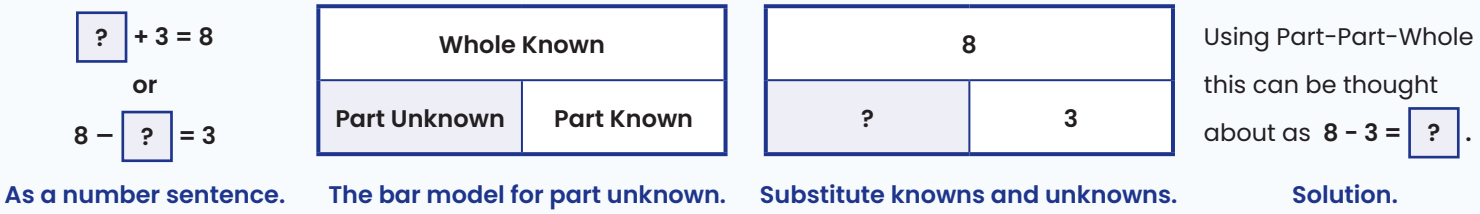


Figure 9. Part Unknown

Whole Unknown

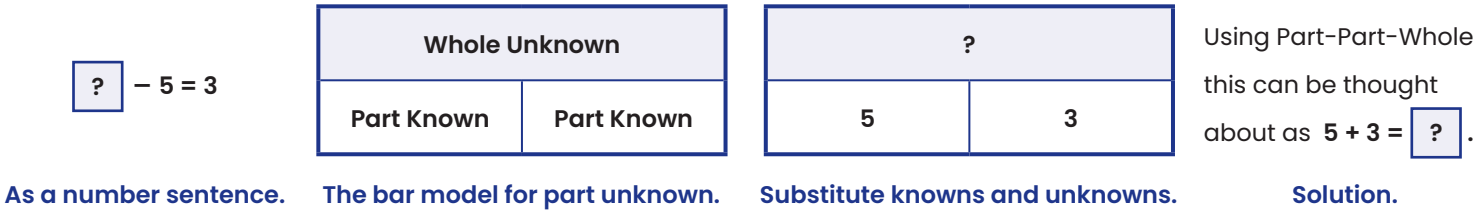
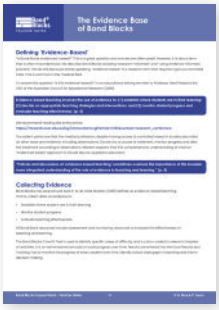


Figure 10. Whole Unknown

This article has modelled the concrete-representational-abstract approach to teaching part-part-whole.



Bond Blocks uses this approach. For more information about the evidence underpinning the system see: **https://bondblocks.com/is-bond-blocks-evidence-based/**

For more information about Part-Part Whole see:

Rice, N and Swan, P. (2022). Building Equations: Using Part-Part-Whole in *Bond Blocks Support Book - Teacher Notes*. https://bondblocks.com/core-teaching-notes

Partition Board

Concrete

Most manipulatives can be counted by ones.

Many students struggle to move from counting to add and subtract at a concrete level, to fluently adding and subtracting at an abstract level.

Bridging the Gap

Representational

Bond Blocks bridge the gap from concrete to abstract because they do not have individually scored lines. Bond Blocks are a representational manipulative because:

- The length and written numeral represent the number of cubes.
- How they are arranged (using part-part-whole) and moved represents the operations of addition and subtraction.

The length base of Bond Blocks makes them self-checking.
Students say the number sentence as they manipulate the physical blocks. Connecting verbal and visual information helps students build connected schemas and effectively store information in their long-term memory.

Abstract

Adding and subtracting using numbers and symbols.

Part + Part = Whole

3 + 2 = 5

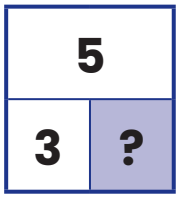
2 + 3 = 5

Whole - Part = Part

5 - 3 = 2

5 - 2 = 3

Extend with algebraic thinking.



5 - ? = 3

3 + ? = 5